

Maximizing Broadcast Tree Lifetime in Wireless Ad Hoc Networks

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Abstract— In wireless ad hoc networks (WANETs), e.g. wireless sensor networks, battery-powered devices are constrained by limited amount of energy. Many WANET applications require that the duration (called *lifetime*) for which the network remains operational - until the first node exhausts its battery energy - is maximized. We study the problem of maximizing the lifetime of WANET broadcast trees under two receiver cost models: 1) the constant receiver power (CORP) model, in which a receiver consumes a fixed amount of energy for receiving an information bit; and, 2) the transmitter-receiver power tradeoff (TREPT) model, in which the amount of energy consumed by a receiver is a function of the received signal power and hence the transmitter power. We propose a graph theoretic solution for CORP model to find a maximum lifetime tree and a binary search based solution for TREPT model to determine power assignment which maximizes the lifetime of a given broadcast tree. Both polynomial-time solutions are formally proved to be optimal.

I. INTRODUCTION

Energy efficiency of Wireless Ad hoc NETWORKS (WANETs) has been an active research area for years. Due to the absence of line power supplies, nodes are constrained by the limited amount of battery energy. In particular, when a network, e.g. a wireless sensor network (WSN), is composed of a large number of nodes, it is impractical to replace all the batteries. Broadcast, an essential primitive in WANETs, is to deliver information from its source to all other nodes using efficient strategies, e.g. broadcast trees. Applications of broadcast include command and query distribution, and software updating.

In this paper, we study the problem of *Maximizing Broadcast Tree Lifetime* (MaxBTL) in WANETs. A variety of definitions of network lifetime is discussed in [1]. We consider the scenario in which each designated receiver is of the same importance, i.e. the network lifetime is the duration until the first node exhausts its battery energy. We assume a broadcast tree is used until the first node (can be a leaf node) in the tree dies and hence the network lifetime is the broadcast tree lifetime. A node can affect the signal strength at its designated receivers as well as choose the set of receivers by adjusting its transmission power. By carefully selecting the transmission power at each node, we can prolong the network lifetime while satisfying connectivity constraints.

We consider two receiver cost models: the *Constant Receiver Power* (CORP) model and the *Transmitter-Receiver Power Tradeoff* (TREPT) model. Specifically, under the CORP

model, a receiver can detect an information bit by consuming a fixed amount of energy. This model was adopted by LEACH (Low-Energy Adaptive Clustering Hierarchy) [2]. While the CORP model is reasonable where the distance is large because the transmission power dominates the overall power consumption in a node in WANETs where nodes are densely distributed (the average distance between nodes is below 10 m), the circuit power consumption along the signal path becomes comparable to or even dominates the transmission power [3]. Vasudevan et al. introduced the TREPT model, for which the energy consumption at the receiver to decode a signal is a function of the transmission energy [4]. Essentially, by increasing the power used by a transmitter to transmit a signal, the decoder, e.g. turbo decoders, can decode the signal faster and expend less energy [4].

We propose two optimal solutions to the MaxBTL problem, including a graph theoretical approach under the CORP model and a binary search algorithm under the TREPT model. The optimality proofs of both algorithms are formally presented. To the best of our knowledge, this is the first work on the maximization of broadcast tree lifetime while considering the tradeoff between transmitter and receiver power.

The rest of the paper is organized as follows. Section II summarizes related work. Section III introduces the power consumption model and formulates the MaxBTL problem. Section IV presents a graph theoretic approach to the MaxBTL problem under the CORP model. Section V proposes a binary search algorithm for the MaxBTL problem under the TREPT model. Finally, Section VI concludes the paper.

II. RELATED WORK

Camerini proved that a Minimum-weight Spanning Tree (MST) minimizes the maximum link weight among all spanning trees in an undirected graph [5]. Its application to WANETs is that, when a WANET is modeled as an undirected graph (i.e. each pair of nodes is connected by an undirected link that is associated with the minimum required transmission power for successful transmission between the two incident nodes), a MST minimizes the maximum transmission power among all broadcast trees [6]. In the special case, where the amount of battery energy of all nodes is identical, the MaxBTL of a node is inversely proportional to its transmission power and hence a MST is a maximum lifetime broadcast tree. Das

et al. extended the result to a network with various battery capacities and proposed a minimum decremental lifetime (MDLT) algorithm [7]. Lloyd et al. and Floréen et al. sought a subnetwork of the maximum lifetime in which the source node is connected to all other nodes [8] [9]. It is easy to see that any broadcast tree contained in such a subnetwork has the maximum lifetime. However, all the above solutions assumed no receiver power and hence cannot be applied to the MaxBTL problem under the CORP or TREPT model. The CORP model was used in L-REMiT (lifetime-refining energy efficiency of multicast trees), a heuristic to the MaxBTL problem, proposed by Wang and Gupta [10], but it is not necessarily optimal. Vasudevan et al. solved the problem of maximizing the lifetime of a data aggregation tree in WSNs under the TREPT model [4]. Since each node in a data aggregation tree transmits the data collected by its descendants and itself to its parent node, the solution cannot be applied directly to the MaxBTL problem which takes advantages of the broadcast nature of the wireless medium.

III. PRELIMINARIES

In this section, we introduce the power consumption model and formulate the optimization problem.

A. Power Consumption Model

In WANETs, a node may act as a transmitter, a receiver or both. The *power consumption* (in Watts) of the transceiver of a node u , denoted by p_u , is the sum of transmission and receiving power consumption denoted by p_u^T and p_u^R , respectively, i.e. $p_u = p_u^T + p_u^R$. In particular, $p_u^R = 0$ if u does not receive any packet from other nodes, e.g. the source node in a broadcast tree; $p_u^T = 0$ if u does not forward any packet to other nodes, e.g. a leaf node in a broadcast tree.

A signal can be successfully detected if the signal strength at the receiver is above certain level after traversing the fading channel. For any pair of nodes u and v , we define $p(u, v)$ as the *transmission power threshold* of u being successfully received by v . In other words, the transmission from u to v necessitates $p_u^T \geq p(u, v)$. This definition is very general and is applicable to networks with asymmetric radio channels and/or heterogeneous transceivers.

In the wireless medium, a single transmission can be received by multiple receivers, which assists in conserving energy and is referred to as Wireless Multicast Advantage [11]. The technique for optimizing a certain goal, e.g. maximizing network lifetime and minimizing power consumption, by adjusting the transmission power is referred to as *power control*. Given a set of designated receivers, the transmission power is dependent on the underlying receiver power consumption model. We summarize the two models used in this paper.

1) *Constant Receiver Power (CORP) Model*: This model assumes that the receiving power, which may vary from node to node, is fixed regardless of the signal strength at the receiver. Therefore, to reach a set C of one-hop neighbors, the transmission power of node u is not lower than the

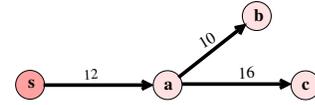


Fig. 1. A broadcast tree rooted at s . An arrow, from a transmitter to a receiver, is associated with the transmission power threshold in mW (milliWatt).

transmission power threshold to any node in C , i.e.

$$p_u^T = \max_{v \in C} p(u, v). \quad (1)$$

In a broadcast tree, a node forwards packets to all its children, if any, that are the immediate downstream nodes along the tree. In Fig. 1, $p_a^T = \max\{p(a, b), p(a, c)\} = 16$ mWs; if $p_a^R = 5$ mWs, $p_a = p_a^T + p_a^R = 21$ mWs.

2) *Transmitter-REceiver Power Tradeoff (TREPT) Model*: Under the TREPT model, the energy consumption of a receiver, say v , for decoding a signal, is a function of the transmission energy of the transmitter, say u , as well as the distance between them, denoted by $d(u, v)$ [4]. For simplicity, we focus on the *mean transmission/receiving power*, i.e.

$$p_v^R = f(p_u^T, d(u, v)^\alpha), \quad (2)$$

where α is the path-loss exponent and $f(\cdot)$ is a monotonic non-increasing function of p_u^T for $p_u^T \geq p(u, v)$. For example, let $p_a^R = \frac{d(s, a)^3}{p_s^T}$ and $d(s, a) = 5$ m in Fig. 1. When $p_s^T = 12$ mW, $p_a^R = \frac{125}{12} = 10.4$ mW; if p_s^T increases to 20 mW, p_a^R reduces to $\frac{125}{20} = 6.25$ mW.

B. Problem Statement

We denote the amount of battery energy of a node u by E_u , which may vary from node to node. A battery is treated as a linear storage of current, i.e. the battery lifetime is the ratio between the initial amount of energy and the discharge current [12]. Since the transceiver is the dominant power consumer during system operation in a node [13], the *lifetime of a node u* (in time units), denoted by ℓ_u , is the ratio between E_u and its power consumption p_u , i.e. $\ell_u \equiv \frac{E_u}{p_u}$. In the rest of the paper, we assume that a node is reliable in the sense that it dies only in the case of depletion of the battery energy.

We denote a broadcast tree (rooted at a node s) by T (T_s). The *lifetime* of T , denoted by $\mathcal{L}(T)$, is the duration until the first node in T fails, i.e.

$$\mathcal{L}(T) \equiv \min_{u \in T^n} \ell_u, \quad (3)$$

where T^n is the set of nodes in T [6] [7] [8] [9]. In this paper, we study the problem of maximizing single-source broadcast tree lifetime (MaxBTL) in stationary WANETs under both the CORP and TREPT models. Specifically, given a WANET W and a node s , we seek a broadcast tree T_s^* such that $\mathcal{L}(T_s^*) = \max_{T \in \mathcal{T}_s} \mathcal{L}(T)$, where \mathcal{T}_s is the set of broadcast trees rooted at s in W . We refer to T_s^* as a *maximum lifetime broadcast tree*, which is not necessarily unique.

IV. MAXBTL SOLUTION UNDER CORP MODEL

In this section, we present an optimal solution to the MaxBTL problem under the CORP model by employing a graph theoretic approach. We first model a WANET as a directed and link-weighted graph and then transform the MaxBTL problem into an optimization problem in the graph.

A. Longevity of Transmitter-Receiver Pair

We defined the broadcast tree lifetime from a so-called *node-based* perspective in (3). To employ a graph theoretic approach, we need to choose a proper metric from a *link-based* perspective. A node u and its receiver v constitute a *Transmitter-Receiver Pair (TRP)*, denoted by $u \rightarrow v$. Intuitively, a broadcast tree is alive until the first TRP breaks. Let $u \rightarrow v$ be a TRP in a tree T_s ; $u \rightarrow v$ breaks when either u or v fails, whichever happens first. Note that, the values of p_u and p_v are specific to a broadcast tree. As a result, the lifetime of a TRP is broadcast tree specific or “dynamic”. Intuitively, this makes the MaxBLT problem more difficult to solve using graph theory. However, we show in the following that an equivalent “static” assignment is possible for TRP values, making the problem solvable by traditional graph theoretic approach.

We define the *longevity* of a TRP $u \rightarrow v$, denoted by $\ell(u, v)$, as

$$\ell(u, v) = \min \left\{ \frac{E_u}{p(u, v) + p_u^R}, \frac{E_v}{p_v^R} \right\}. \quad (4)$$

$p_u^R = 0$ if u is the source node. This definition excludes the effects of all transmissions incident to u and v other than the one from u to v . In Fig. 1, for example, TRP $a \rightarrow c$ has no effects on $\ell(s, a)$ and $\ell(a, b)$. Note that we choose the term “longevity” to distinguish it from the literal meaning of “lifetime”. In the following, we verify the appropriateness of the definition of the TRP longevity by showing that the broadcast tree lifetime is the minimum longevity of any TRP.

Lemma 1: In any broadcast tree T , $\mathcal{L}(T) = \min_{u \rightarrow v \in T^t} \ell(u, v)$, where T^t is the set of TRPs in T .

Proof: We denote the set of nodes and the set of leaf node in T by T^n and T^ℓ , respectively. Note, $T^\ell \subseteq T^n$. In (5) below, the transmission power threshold is replaced by nodal transmission power for all intermediate nodes due to (1); all non-leaf nodes are removed from the last term because they are considered as intermediate nodes.

$$\begin{aligned} \min_{u \rightarrow v \in T^t} \ell(u, v) &= \min_{u \rightarrow v \in T^t} \left\{ \frac{E_u}{p(u, v) + p_u^R}, \frac{E_v}{p_v^R} \right\} \\ &= \min \left\{ \min_{u \rightarrow v \in T^t} \frac{E_u}{p(u, v) + p_u^R}, \min_{u \rightarrow v \in T^t} \frac{E_v}{p_v^R} \right\} \\ &= \min \left\{ \min_{u \in T^n \setminus T^\ell} \frac{E_u}{p_u^T + p_u^R}, \min_{v \in T^\ell} \frac{E_v}{p_v^R} \right\} \quad (5) \\ &= \min \left\{ \min_{u \in T^n \setminus T^\ell} \ell_u, \min_{v \in T^\ell} \ell_v \right\} \\ &= \min_{u \in T^n} \ell_u = \mathcal{L}(T). \end{aligned}$$

B. Inverse TRP Longevity Graph

We model a WANET as a directed and link-weighted graph $G = (N, A, w)$ called an *INverse TRP longevity Graph (ING)*, where N is a set of nodes, A is a set of links and $w : A \rightarrow \mathbb{R}^+$ is a weight function. For each pair of nodes $u, v \in N$, there is a link in A joining u and v , denoted by (u, v) , if $\mathcal{P}_u^T \geq p(u, v)$, where \mathcal{P}_u^T is a node-dependent constraint called *maximum transmission power*. A node u is capable of adjusting its transmission power up to \mathcal{P}_u^T . The function w assigns each link $(u, v) \in A$ the value of $\frac{1}{\ell(u, v)}$, i.e. the inverse longevity of TRP $u \rightarrow v$. By modeling a WANET as an ING, according to Lemma 1, we transform the MaxBTL problem into deriving a rooted spanning tree that minimizes the maximum link weight. While broadcast tree is a term used in computer networks, *spanning tree* is its counterpart in graph theory.

C. An Optimal Graph Theory Based Solution

We show that, in a directed graph, a rooted spanning tree formed by using Prim’s algorithm [14] minimizes the maximum link weight among all spanning trees rooted at the same node in Lemma 2. Then, we propose a graph theoretic approach, MBL-CORP (see Algorithm 1), to the problem of MaxBTL: first, a WANET is modeled as an ING; then, a spanning tree that is rooted at the given node is formed by running Prim’s algorithm [14]. We prove in Theorem 1 that, under the CORP model, Algorithm 1 results in an optimal solution to the problem of MaxBTL.

Lemma 2: In graph $G = (N, A, w)$, where $w : A \rightarrow \mathbb{R}^+$, if \mathcal{T}_s is the set of spanning trees rooted at a node $s \in N$ and $T_s^* = (N, E \subseteq A) \in \mathcal{T}_s$ is formed using Prim’s algorithm,

$$\max_{(i, j) \in E} w(i, j) = \min_{T_s = (N, A' \subseteq A) \in \mathcal{T}_s} \max_{(i, j) \in A'} w(i, j). \quad (6)$$

Proof: Prim’s algorithm builds a tree that starts as a single node s . In each iteration that follows, it grows the tree by adding the lightest link that starts from a tree node and ends in a non-tree node. The process repeats until all the nodes in the graph are included in the tree. We denote the node chosen in the k^{th} iteration and the corresponding intermediate tree by i_k and $T_s^{(k)}$, respectively ($1 \leq k \leq n = |N|$). Note, $T_s^{(n)} = T_s^*$. Further, let a *path* $\pi(u, v)$ be a sequence of unrepeated nodes from nodes u to v , in which any two successive nodes u_i and u_{i+1} are joined by the link (u_i, u_{i+1}) . If *each* link of a path belongs to a tree, the path is *in* the tree.

Assume a tree $T_s' = (N, E' \subseteq A) \in \mathcal{T}_s$ such that

$$w(u, v) = \max_{(i, j) \in E} w(i, j) > \max_{(i, j) \in E'} w(i, j). \quad (7)$$

Let (u, v) belong to a path $\pi(s, d)$ ($d \in N \setminus \{s\}$) in T_s^* and $\pi'(s, d)$ be a path in T_s' (see Fig. 2). (u, v) does not belong to $\pi'(s, d)$ due to (7). We backtrack to the iteration, say h ($2 \leq h \leq n$), when (u, v) was chosen. Since $\pi'(s, d)$ is not in T_s^* , it is not in $T_s^{(h)}$. Let (u', v') be the first link in $\pi'(s, d)$ that does not belong to $T_s^{(h)}$. Since (u, v) was picked over (u', v') in the h^{th} iteration, $w(u, v) \leq w(u', v')$ per Prim’s algorithm, but it contradicts (7). Hence, T_s' does not exist, i.e.

$$\max_{(i, j) \in E} w(i, j) \leq \min_{T_s = (N, A' \subseteq A) \in \mathcal{T}_s} \max_{(i, j) \in A'} w(i, j). \quad (8)$$

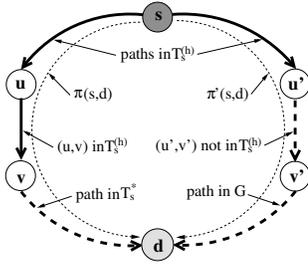


Fig. 2. Lemma 2. A solid arch/arrow represents a path/link in $T_s^{(h)}$. A dotted and bold arch/arrow is a path/link not in $T_s^{(h)}$.

On the other hand, because $T_s^* \in \mathcal{T}_s$,

$$\max_{(i,j) \in E} w(i,j) \geq \min_{T_s = (N, A' \subseteq A) \in \mathcal{T}_s} \max_{(i,j) \in A'} w(i,j). \quad (9)$$

We obtain (6) by combining (8) and (9). ■

Algorithm 1 MBL-CORP

Input: A WANET W and a node s

Output: A broadcast tree

- 1: $G \leftarrow \text{Model}(W)$ {Model a WANET as an ING}
 - 2: $T_s \leftarrow \text{Prim}(G,s)$ {Prim's algorithm [14]}
 - 3: **return** T_s
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Theorem 1: Under the CORP model, Algorithm 1 builds a maximum lifetime broadcast tree rooted at the given node in $O(m \log n)$ time, where n and m are the numbers of nodes and TRPs in the WANET, respectively.

Proof: By modeling a WANET as an ING, the problem of MaxBTL is transformed into seeking a rooted spanning tree whose maximum link weight is minimized in the ING. Lemma 1 shows the two problems are equivalent. Then, Lemma 2 proves that Prim's algorithm provides optimal solutions to the corresponding problem in the graph. Hence, Algorithm 1 is an optimal solution to the MaxBTL problem.

The time complexity of *Model* and *Prim* in a graph with n nodes and m TRP pairs/links is $O(m)$ and $O(m \log n)$, respectively. So, Algorithm 1 runs in $O(m \log n)$ time. ■

V. MAXBLT SOLUTION UNDER TREPT MODEL

A. Power Setting of a Broadcast Trees

A *power setting* is a snapshot of p_u^T and p_u^R of each node u in a broadcast tree. Assume a broadcast tree with predefined lifetime τ . We observe that the tree lifetime does not change if a node u consumes more energy than necessary as long as its lifetime $\ell_u \geq \tau$. Therefore, given a broadcast tree and desirable lifetime, we can detect the existence of a power setting by checking if the connectivity constraint is violated when letting each node deplete its battery energy right after the given lifetime duration. This can be done in a top-down process, where the power setting is computed hop by hop from the source node to leaf nodes along the broadcast tree.

We clarify the idea by using an example in which we assume the broadcast tree depicted in Fig. 1 and examine the existence of a power setting, given lifetime τ . The process

starts from the source node s . Since s only consumes power for transmitting, $p_s^T = \min \{ \frac{E_s}{\tau}, \mathcal{P}_s^T \}$, where \mathcal{P}_s^T is the maximum transmission power. If $p_s^T < p(s,a)$, TRP $s \rightarrow a$ breaks and the tree is disconnected. The process terminates whenever a broken TRP is detected, which means no power setting exists. Otherwise, the process continues with the succeeding nodes, if any. Knowing p_s^T, p_a^R can be computed by using (2) and $p_a^T = \min \{ \frac{E_a}{\tau} - p_a^R, \mathcal{P}_a^T \}$. We then compare p_a^T against $p(a,b)$ and $p(a,c)$. If all TRPs in the tree are connected, we find the power setting.

We summarize the procedure of examining the existence of any power setting of a given broadcast tree against the desirable lifetime in the function *Feasible* (see Algorithm 2). *Feasible* takes as input a WANET W , a broadcast tree T_s and predefined lifetime τ ; it returns *true* if a power setting exists to achieve lifetime τ or *false* otherwise. We denote a queue of nodes as Q ; each node u in Q is covered by the broadcast tree, but no child of u is covered by the tree. Initially, s is the only node in Q . In each iteration of the *while* loop in lines 2 through 15, a node, say u , is removed by *Dequeue* from Q . Node u first computes its transmission power based on its receiver power by taking into account the lifetime constraints and its maximum transmission power. When a negative transmission power is obtained, it means the receiver power alone consumes too much power to satisfy the lifetime constraint. For each of u 's child node in T_s , say v , if v can be covered by u 's transmission, i.e. $p_u^T \geq p(u,v)$, then v computes its corresponding receiver power according on the transmission power of its parent node, i.e. node u , based on (2) before being included in Q ; otherwise, the function returns *false* indicating the lifetime τ is not feasible because TRP $u \rightarrow v$ in T_s breaks. The *while* loop continues until an unreachable node is found, in which case the function returns *false*, or Q is empty, in which case each node in the network W is covered by T_s and the function returns *true*.

Algorithm 2 Feasible

Input: A WANET W , a broadcast tree T_s and lifetime τ

Output: *true/false*

- 1: $p_s^R \leftarrow 0$; *Enqueue*(Q,s)
 - 2: **while** ($u = \text{Dequeue}(Q)$) \neq null **do**
 - 3: $p_u^T \leftarrow \min \{ \frac{E_u}{\tau} - p_u^R, \mathcal{P}_u^T \}$
 - 4: **if** $p_u^T < 0$ **then**
 - 5: **return false**
 - 6: **end if**
 - 7: **for all** node v s.t. $u \rightarrow v \in T^t$ **do**
 - 8: **if** $p_u^T \geq p(u,v)$ **then**
 - 9: $p_v^R \leftarrow f(p_u^T, d(u,v)^{\alpha})$
 - 10: *Enqueue*(Q,v)
 - 11: **else**
 - 12: **return false**
 - 13: **end if**
 - 14: **end for**
 - 15: **end while**
 - 16: **return true**
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B. Binary Search Algorithm

We propose a binary search algorithm (see Algorithm 3) to the MaxBTL problem under the TREPT model. That is, given a broadcast tree in a WANET, we seek the maximum lifetime of the tree. It is clear that the maximum broadcast tree lifetime is bounded by any node lifetime in the tree, particularly the source node's lifetime. Given a broadcast tree T_s , the source node s has to transmit packets to all its children. So, the upper-bound lifetime is $\tau^u = \min_{s \rightarrow v \in T_s^t} \frac{E_s}{p(s, v)}$, where T_s^t is the set of TRPs in T_s . On the other hand, we set the lower-bound lifetime $\tau^l = 0$ since any node in the tree can have the lower-bound lifetime. In line 1 of Algorithm 3, the lifetime bounds are initialized. In each iteration of the *while* loop in lines 2 through 9, the current lifetime τ is updated to the median of the two bounds; the lower-bound increases to τ if τ is feasible, otherwise the upper-bound decreases to τ . The loop ends when the difference between the two bounds is smaller than the given margin ϵ . Algorithm 3 returns the maximum lifetime of the given broadcast tree.

Algorithm 3 Binary Search Algorithm

Input: A WANET W , broadcast tree T_s and error-margin ϵ

Output: The maximum lifetime of T_s

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1:  $\tau^u = \min_{s \rightarrow v \in T_s^t} \frac{E_s}{p(s, v)}$ ;  $\tau^l \leftarrow 0$ 
2: while  $\tau^u - \tau^l > \epsilon$  do
3:    $\tau \leftarrow \frac{\tau^u + \tau^l}{2}$ 
4:   if Feasible( $W, T_s, \tau$ ) then
5:      $\tau^l \leftarrow \tau$ 
6:   else
7:      $\tau^u \leftarrow \tau$ 
8:   end if
9: end while
10: return  $\tau$ 

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We prove the optimality of the binary search algorithm in the following theorem.

Theorem 2: Under the TREPT model, Algorithm 3 returns the lifetime of any given broadcast tree T_s within ϵ of the optimal lifetime in $O(n \log \frac{\tau^u}{\epsilon})$, where n is the number of nodes in the WANET and $\tau^u = \min_{s \rightarrow v \in T_s^t} \frac{E_s}{p(s, v)}$.

Proof: It is clear that the function *Feasible* in Algorithm 2 is a monotonic function: Given a broadcast tree T_s in a WANET W , if for the lifetime τ , the function returns *true* (*false*), then for any value above (below) τ , it returns *true* (*false*). So, during the execution of the binary search algorithm, the maximum lifetime is always bounded by the lower- and upper-bound lifetime. The *while* loop in Algorithm 3 terminates when the difference between the two bounds is smaller than ϵ . Therefore, the lifetime of the given broadcast tree is within ϵ of the maximum lifetime.

We analyze the time complexity. Initially, the difference between the two lifetime bounds is τ^u . So, the *while* loop in Algorithm 3 repeats $O(\log \frac{\tau^u}{\epsilon})$ times because binary search is

used. In *Feasible*, the receiver power and transmission power are computed exactly once for each node in the network. So, its time complexity is $O(n)$. Consequently, the total running time of the binary search algorithm in Algorithm 3 is $O(n \log \frac{\tau^u}{\epsilon})$. ■

So far, we have solved the MaxBTL problem under the TREPT model when a broadcast tree is given. We suspect that the general problem of finding a maximum lifetime broadcast tree under TREPT model is NP-hard.

VI. CONCLUSIONS AND FUTURE WORK

The lifetime of a broadcast tree is the duration until the first node in the tree fails due to battery energy exhaustion. In this paper, we presented optimal solutions to the problem of MaxBTL in stationary WANETs. Specifically, a graph theoretical approach was proposed in the CORP model and a binary search algorithm in the TREPT model. The solutions were proved to be optimal. Future directions include the general MaxBTL problem under the TREPT model and distributed solutions under both models.

VII. ACKNOWLEDGMENTS

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