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Energy-Efficient Multicast Protocols

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1 Introduction

The goal of a *multicast* communication (also known as one-to-many/many-to-many communication) service is to efficiently deliver messages originating from a single or multiple sources to multiple recipients. Multicast communication primitives are useful in many distributed systems and applications where multiple parties are involved in tasks such as collaboration, resource and/or information sharing, and co-ordination. In this chapter we are mainly interested in applications which involve distribution of same information to multiple recipients from a single source or multiple sources. A multicast can be viewed as a *selective broadcast* in the sense that only a subset of the system may be interested in receiving the broadcast messages. Although a simple way to implement a multicast operation is to use system wide broadcast and do filtering at the end systems, this can be a significant waste of resources, such as network bandwidth and processing. The degree of inefficiency depends upon the sparsity of the interested systems. At the other extreme a multicast can be implemented by sending a unicast message to each interested system. This may be efficient way to implement multicast for a sparse distributed group but it is a severe waste of resources for moderately sized groups since the message may need to unnecessarily travel a common portion of path to multiple receivers several times wasting bandwidth. This also increases the latency of the multicast message delivery by sequentializing the message delivery to multiple recipients. In systems where bandwidth is at premium and/or low latency is desired multicast services employ multicast delivery structures such as a multicast tree (or mesh for better fault tolerance). The problem of building an optimal multicast tree is akin to the widely known NP-complete Steiner tree problem [1, 2]. Since efficient implementation of multicast operation has the potential to significantly improve application or system performance, understandably many heuristic protocols exist to construct efficient multicast distribution structures.

In this chapter we are mainly concerned with multicast in Wireless Ad hoc NETworks (WANETs). In such networks battery-powered nodes, in addition to performing processing functions, route messages hop-by-hop using wireless transceivers. The optimization goal for constructing a multicast distribution structure in a WANET becomes even more challenging. Not only the bandwidth consumption needs to be optimized to conserve the scarce and shared wireless bandwidth, but also, and perhaps more importantly, consumption of battery power (or energy) (which in some cases, e.g. in some sensor networks, is non-renewable) needs to be minimized in order to maximize the *lifetime* of the network - the duration for which sufficient number of nodes in the system have ample energy to provide the desired service. In case the nodes are also mobile the applicability of maintaining a multicast distribution structure is contingent upon the cost of maintaining the distribution structure in face of disruption due to frequent mobility of nodes.

In this chapter, we present some solutions for constructing energy-efficient multicast distribution trees for WANETs. We begin with describing general techniques for conserving energy needed for communication and describe metrics for modeling cost of energy consumption which can be used to guide optimization of energy consumption for multicasting in wireless networks.

2 Energy-Efficient Communication Techniques

Compared with wired device, one of the greatest limitations of wireless devices is finite power supplies. Thus, any form of communication involving wireless devices need to be as energy-efficient as possible. In this section, we discuss some basic techniques for energy-efficient communication in WANETs. Further, we introduce the basic metrics and cost models used in constructing energy-efficient multicast distribution trees.

2.1 Energy Consumption Model for Wireless Communication

There are three major causes of energy depletion in a node: 1) energy expended for RF propagation; 2) energy expended in the transmitting hardware for operation such as encoding and modulation; and 3) energy expended in the receiving hardware for operations such as demodulation and decoding. For simplicity, we assume that the energy expended for transmission and reception is the same for all the nodes in the system and denote them by E^T and E^R, respectively. We neglect any energy consumption that occurs when the node is simply "on" (idling), although it would be easy to incorporate it into this model. The minimum energy cost (per bit) needed by node i to transmit a packet to an adjacent node j, $E_{i,j}$, is modeled as:

$$E_{i,j} = E^T + K(\max\{p_{min}, r_{i,j}^{\alpha}\}), \qquad (1)$$

where $r_{i,j}$ is the Euclidean distance between i and j, K is a constant dependent upon the properties of the antenna, and α is a constant which is dependent on the propagation losses in the medium, and p_{min} is the minimum transmission power required to send message to an arbitrarily near node (this accounts for the fact that the

$r^{-\alpha}$ dependence is only in the far-field region of the transmitting antenna) [3][4]. Depending upon the relative contribution from the distance-dependent and distance independent energy consumption factors, the radios can be categorized as: *long range radios* and *short range radios*. For long range radios, E^T is much smaller than E^{RF}_{max} (E^{RF}_{max} is the energy cost (per bit) for a node using maximum transmission power). But for short range radios (such as sensor networks), the distance independent part is the dominant factor e.g. $E^T \approx 4E^{RF}_{max}$ for some short range radios [5, 6]. A node can control its communication range by controlling the transmission power. Hence, the connectivity of network depends on the transmission power. We use wireless power control model in [6]: Each node can choose its power level p, where $0 \leq p \leq p_{max}$.

2.2 Basic Techniques for Conserving Energy

In general, there are four basic techniques for energy-efficient communication [7].

1. The first technique is to turn-off non-used transceivers to conserve energy e.g. PAMAS protocol [8].

2. The second technique is scheduling the competing nodes to avoid wastage of energy due to contention. This can reduce the number of retransmission and increase nodes' lifetime by turning off the non-used transceivers for a period of time. For example, a base station in a infrastructure based wireless network can broadcast a schedule that contains data transmission starting times for each node as in [9].

3. The third technique is to reduce communication overhead, such as defer transmission when the channel conditions are poor [10].

4. The fourth technique is to use power control to conserve energy. The transmit power P_t needed to reach a node at a distance of d is proportional to d^α, where $\alpha(\geq 2)$ is a transmission medium dependent constant. Hence, a node can adjust its transmission power to a level which is sufficient to reach the receiving node. This has the added advantage of reducing interference with other on-going transmissions.

In this chapter, we will mainly discuss the approaches using power control technique for energy-efficient multicast.

2.3 Wireless Multicast Advantage

A network-wide multicast distribution tree is a spanning tree which "covers" all the multicast group nodes. A *source-based multicast tree* is rooted at a source node.

On the other hand a *group-shared tree* is rooted at a *core node* (also known as rendezvous node) [11, 12]. Although all the leaf nodes in a multicast tree are necessarily multicast group nodes, the intermediate nodes (called *forwarding nodes*) may or may not be multicast group nodes. The message is distributed to all the multicast group members by flooding it on the distribution tree, i.e. the source node forwards its message to all its tree neighbors (children in a source based tree) and a forwarding node forwards the message to all its tree neighbors except the one from which it received the message. Hence, the flooding of a message on the multicast tree gets broken down into several *local forwarding* operations.

The energy consumed for a local forwarding operation (as well as the connectivity in the wireless network) depends largely on the transmission power at the nodes. Assume that each node can dynamically select its transmission power level p^{RF}, where $0 \leq p^{RF} \leq p_{max}$. Let $p_{i,j}^{RF}$ be the minimum power needed to transmit a packet over the link between nodes i and j. The power level information can be obtained from the link layer using power-control techniques [13]. Therefore, the total power expenditure of node i, when forwarding a packet to another node j, $p_{i,j}$ is:

$$p_{i,j} = \begin{cases} p_{i,j}^{RF} + p^T & \text{if } i \text{ is the source node,} \\ p_{i,j}^{RF} + p^T + p^R & \text{otherwise.} \end{cases} \tag{1}$$

Since a leaf node in a multicast tree only receives data without forwarding it further to any other node, its power expenditure is simply equal to p^R in our model.

The power consumption model can be transformed to energy consumption model by introducing time. We use τ to denote the time period for a node to transmit a bit. We assume that this quantity is the same at all the nodes. Then, the total energy (per bit) expenditure of node i, when forwarding a packet to node j, is

$$E_{i,j} = \begin{cases} E_{i,j}^{RF} + E^T & \text{if } i \text{ is source node;} \\ E_{i,j}^{RF} + E^T + E^R & \text{otherwise,} \end{cases} \tag{2}$$

where $E_{i,j}^{RF}$ is the energy cost (per bit) of the link between nodes i and j for a packet transmission, $E_{i,j}^{RF} = \tau p_{i,j}^{RF}$, E^T is energy cost (per bit) of electronics and digital processing, and E^R is energy cost (per bit) at the receiver side. In this chapter, we use $E_{i,j}^{RF}$ as the energy cost of wireless link from node i to node j. Also we assume that link costs are symmetric, i.e. $E_{i,j}^{RF} = E_{j,i}^{RF}$.

The local forwarding operation can exploit the broadcast property of wireless communication. This not only may conserve bandwidth (the message needs to be forwarded only once) and may reduce latency (the message can be simultaneously forwarded to multiple nodes), but also presents an opportunity to conserve energy. For example, consider that a (forwarding or source) node i needs to forward data to its neighbors, say, nodes j and k (see Figure 1) which may be its tree neighbors in a multicast tree. Then, by transmitting data at the power level $\max\{p_{i,j}, p_{i,k}\}$, node i can simultaneously send packets to both nodes j and k. Compared to wired networks, this reduces power consumption for forwarding in a wireless network from sum of power consumed for each forwarding link ($p_{i,j} + p_{i,k}$) to maximum of power required over all the forwarding links ($\max\{p_{i,j}, p_{i,k}\}$). This resource (bandwidth, energy, and time) saving advantage of wireless networks over wired networks is termed as **wireless multicast advantage** [3].

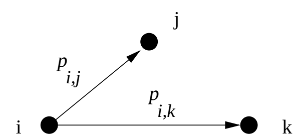

Figure 1: Wireless Multicast Advantage. $p_{i(j,k)} = \max\{p_{i,j}, p_{i,k}\}$, instead of $p_{i,j} + p_{i,k}$.

In summary, wireless medium is a broadcast medium, and one time local transmission can possibly reach all the neighbors. Power control allows a node to conserve energy by transmitting a packet at power level which is sufficient to reach only its tree neighbors (as opposed to all the neighbors) except the one it received the message from. Further, nodes which use more power can reach more nodes reducing the number of forwarding nodes needed to reach all the multicast group members (and hence the network wide multicast tree may have smaller depth). However there is a flip side to this. The node itself will consume energy at a greater rate, hence dieing (running out of energy) faster. Further, more the transmission power used by a node more interferences it causes with other simultaneous transmission[1]. This will reduce the number of simultaneous transmission. We discuss this in more detail in the next section.

2.4 Why Link-Based View is Not Suitable for WANETs?

For simplicity, consider a single source multicast is used to reach a subset of nodes in the network from a given source s. One way to achieve this is that node s simply increases its transmission range to such an extent that it can reach all the group members. In general this "single hop" approach is not universally applicable since nodes usually have a limited maximum range and multiple hops are needed to reach all the nodes in the multicast group. Further, as we will see later, use of multiple

[1]We are assuming a single wireless communication channel is used by all the nodes. Further, we assume that medium access techniques is employed by a node to get access to the channel.

hops may lead to savings in overall energy consumption for the multicast message distribution.

Since a multi-hop solution is needed, the problem becomes that of constructing a minimum cost multicast tree. In wired networks, multicast trees are built based on link costs. However, no predefined “links” exist in a wireless network. Specially, the ones which employ power control. The number of links incident on a node depends upon the power level at which the node transmits. Simply, assuming that a node always transmits at its maximum transmission power may result in too many links and a non-optimal solution. Further, due to broadcast nature of wireless transmission a single transmission may reach multiple nodes. This makes it difficult to assign a “link cost” to each link since now one has to figure out how many nodes were reachable in a single transmission. This in turn depends upon the power level employed by each node and the node distribution.

Based on the above reasons, it’s more appropriate to take a node-based view of wireless networks, where in costs are only assigned to the nodes. Consequently, the problem of constructing an energy-efficient multicast tree becomes equivalent to constructing a multicast tree with *minimum/maximum summation of node cost*. We will present the existing algorithms for constructing energy-efficient multicast trees in Section 4. In the next section, we present the node costs which are suitable for constructing energy-efficient multicast trees.

3 Energy Metrics and Cost Models

In general, there are two different criteria used for energy optimization:

1. **Total Energy Consumption (TEC):** is the total energy consumption of the system to complete a given task.

2. **System Lifetime (SL):** is the volume of task that can be completed with a given energy level in the system.

For example, consider the WANET shown in Figure 2 which consists of three nodes labeled 1, 2, and 3. Assume that all the three nodes belong to a multicast group with node 1 as the source node. Depending upon whether the energy optimization goal is TEC or SL, as illustrated in Figures 3 and 4, the source based multicast tree optimizing these goals is different. Figure 3 shows the multicast tree which minimizes TEC and Figure 4 shows the multicast tree that maximizes SL. Minimum overall energy consumption is achieved by node 1 simply forwarding multicast messages to both nodes 2 and 3 using one local forwarding at the cost of 10 **energy units (EU)**[2] per packet. However, assuming that each node initially has 200

[2]EU can be replaced by appropriate energy units such as J (Joules) or mJ (milli Joules).

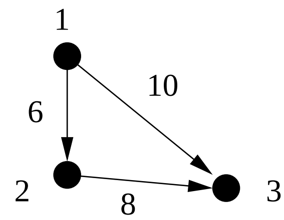

Figure 2: A topology of wireless network with 3 nodes where node 1 is the source node. The figure shows all the existing wireless links. Labels associated with the edges are the average energy cost of a node for transmitting packets over the link, such as energy cost for node 1 transmitting packet to node 2 is 6 EU/packet. Assume the residual battery energy at all three nodes is 200 EU.

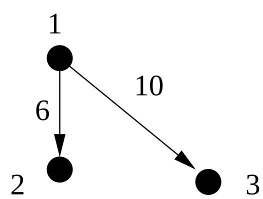

Figure 3: Minimum Energy multicast tree generated from Figure 2. Energy cost of the tree = $\max\{6, 10\} = 10$ EU/packet. Lifetime of the tree = $\frac{200}{10} = 20$ packets.

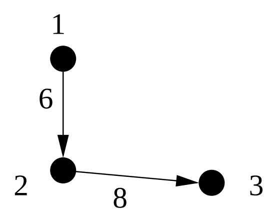

Figure 4: Maximum Lifetime multicast tree generated from Figure 2. Energy cost of the tree = $6 + 8 = 14$ EU/packet. Lifetime of the tree = $\min\{\frac{200}{6}, \frac{200}{8}\} = 25$ packets.

EU of residual battery energy, this strategy is not optimal for optimizing SL, since node 1 can forward only 20 multicast packets (200 EU $\div$ 10 EU/packet) before it dies. Figure 4 shows the multicast tree which has a better lifetime since it can be used to forward 25 packets instead of just 20 packets.

3.1 Node Cost

The definition of node cost depends upon the optimization goals: minimizing energy consumption or maximizing multicast tree lifetime, as well as the type of multicast tree being used: source-based or group-shared. For minimizing energy consumption, node cost is the node energy cost, i.e. average energy cost (in terms of packets) of a node. Let E_i denote the average energy cost of node i in a multicast tree for one multicast packet transmission. For maximizing multicast tree lifetime, the expected remaining "life" of the node itself should be used as node cost. The **lifetime of a node** in a multicast tree, is the maximum number of multicast packets that may be processed (transmitted and/or received) by the node. If we use R_i to denote the residual battery energy of node i, then the lifetime of node i in a multicast tree $LT_i = \frac{R_i}{E_i}$.

3.1.1 Node Cost in Source-based Multicast Tree

In this section we restrict the discussion to calculation of a single node's energy cost for source-based multicast trees. As we have seen earlier, a node's lifetime can be derived from the node's energy cost and its residual battery energy. Let E_i^{RF} be the maximum energy cost (per bit) of the link between nodes i and its children for a multicast packet transmission. In a source-based tree a source node does not incur any reception cost and leaf nodes don't incur any transmission cost, however, a forwarding node incurs both the cost. Hence, node i's energy cost in a source-based multicast tree, E_i, can be modeled as follows:

$$E_i = \begin{cases} E_i^{RF} + E^T + E^R & \text{if node } i \text{ is not a leaf node;} \\ E_i^{RF} + E^T & \text{if node } i \text{ is the source node;} \\ E^R & \text{otherwise.} \end{cases} \quad (3)$$

In Section 4, we will discuss how BIP/MIP [3], EWMA [14], S-REMiT [15] algorithms use Equation 3 as node energy cost to construct minimum-energy source-based multicast trees.

Considering reliable multicast, link error rate needs to be incorporated into the node energy cost. Let e_i be the link error rate for node i to forward the multicast packet to all of node i's children reliably, and e_{i_parent} be the link error rate of node i's parent to forward the packet to all of its own children. The average number of retransmissions to achieve a successful transmission for a link with error probability e can be modeled as $\frac{1}{1-e}$ under the assumption that independent retransmission failures are independent of each other. Hence, Equation 3 can rewritten as follows to incorporate the energy overhead of retransmissions:

$$E_i = \begin{cases} \frac{E_i^{RF}+E^T}{1-e_i} + \frac{E^R}{1-e_{i\_parent}} & \text{if node } i \text{ is not a leaf node;} \\ \frac{E_i^{RF}+E^T}{1-e_i} & \text{if node } i \text{ is the source node;} \\ \frac{E^R}{1-e_{i,parent}} & \text{otherwise.} \end{cases} \quad (4)$$

RBIP algorithm [16] uses Equation 4 as node energy cost to construct minimum-energy source-based multicast trees.

3.1.2 Node Cost in Group-shared Multicast Tree

In contrast to source-based multicast tree, node's energy cost in a group-shared multicast tree is not only decided by the tree links attached to node but also decided by where message is coming from. For example, in Figure 2, if multicast packet coming from node 1, energy cost for node 2 to forward the packet is 8 EU/packet. If the multicast packet coming from node 3, energy cost for node 2 to forward the packet is 6 EU/packet. Assume the message generation rates at nodes 1 and 3 are 7 packets/second and 13 packets/second, respectively. Then the average energy cost of node 2 in the group-shared tree is $\frac{8*7+6*13}{7+13} = 6.7$ EU/packet. So node's cost in group-shared multicast tree needs to incorporate message generation rates of all the multicast source nodes in the group.

We use terminal node to denote a node whose degree on the tree is equal to 1. Assume that the tree links that are incident on a node are ordered in descending order of their energy cost. We use $L_i[s]$ to denote the s-th highest energy cost tree links incident on node i and $E_i[s]$ to denote the energy cost (per bit) of $L_i[s]$. If $L_i[s]$ does not exist, then $E_i[s] \equiv 0$. Suppose L_i is the branch on which the multicast message arrives to be forwarded by node i. Based on the energy cost model in Section 3.1, the energy cost (per bit) of node i, EC_i, for forwarding message arriving on its incident branch L_i in multicast tree T is:

$$E_i = \begin{cases} E^T + E_i[2] + E^R & \text{if } L_i = L_i[1] \text{ and node } i \text{ is not a terminal node} \\ E^R & \text{if } L_i = L_i[1] \text{ and node } i \text{ is a terminal node} \\ E^T + E_i[1] & \text{if } L_i \neq L_i[1] \text{ and } i \text{ is the source node} \\ E^T + E_i[1] + E^R & \text{otherwise} \end{cases} \tag{5}$$

If a multicast message arrives on branch $L_i[1]$ and node i is not a terminal node, the energy cost of i to forward this message is $E^T + E_i[2] + E^R$; however, if i is a terminal node, the energy cost of i is only the reception energy cost E^R. Otherwise, if a multicast message arrives on a branch other than branch $L_i[1]$ and i is not the multicast source, the energy cost of node i to forward this message is $E^T + E_i[1] + E^R$. In the cases that node i is the multicast source, the energy cost of i is just the cost for sending the multicast message to all of its connected tree neighbors, that is $E^T + E_i[1]$.

3.2 Multicast Tree Cost

Similar to node cost, multicast tree cost is also dependent on the above two optimization goals. Note that these two different optimization goals may conflict with each other. Minimum energy consumption multicast tree may result in rapid depletion of energy at intermediate nodes possibly leading to network partition and interruption of the multicast service. On the other hand, a maximum lifetime multicast tree may not include all of the minimum energy routes. For example, Figure 2 is the network topology of three nodes, and Figure 3 and Figure 4 are the minimum energy and maximum lifetime multicast tree, respectively. The two different multicast tree costs are defined as follows:

3.2.1 Minimizing Energy Consumption:

The Total Energy Consumption (TEC) by all the transceivers in the multicast tree T for multicasting one packet is,

$$TEC(T) = \sum_{i \in T} E_i.$$

Using this definition, we can describe the problem of minimizing energy consumption of multicast tree as follows: Let T_G denote the set of all possible multicast trees for a fixed multicast group G. The minimum energy consumption multicast tree T^* is:

$$T^* = \arg \min_{\forall T \in T_G} \{TEC(T)\} = \arg \min_{\forall T \in T_G} \{\sum_{\forall i \in T} E_i\}.$$

This cost requires to carefully choose intermediate transceivers such that the overall energy consumed for the multicasting packet is minimized. Optimizing this cost has been proven to be a NP-complete problem [1, 2]. It is difficult to achieve this cost as it is not easy to select the appropriate intermediate nodes.

3.2.2 Multicast Tree Lifetime

Multicast tree lifetime can be defined as the time duration starting from beginning of multicast service until the first node in the network fails due to battery energy exhaustion. This cost is very important for some critical applications, such as battlefield ad hoc networks. The lifetime of multicast tree is :

$$LT(T) = \min_{i \in T} LT_i.$$

Using this definition, the problem of maximizing lifetime of a multicast tree can be stated as follows: Let T_G denote the set of all possible multicast trees for a fixed multicast group G. The maximum lifetime multicast tree $T^\diamond$ is:

$$T^\diamond = \arg \max_{\forall T \in T_G} \{LT(T)\} = \arg \max_{\forall T \in T_G} \{\min_{\forall i \in T} \frac{R_i}{E_i}\}$$

However, optimizing for this cost is also very difficult, because we need to select the nodes involved in multicasting in such a way that the energy of all the nodes are depleted uniformly. In other words, given a network, we need to determine some nodes that are bottleneck nodes meaning that their energy gets depleted faster than other nodes. The lifetime of the bottleneck nodes determine the lifetime of the entire multicast tree. So optimizing this cost is equivalent to maximizing the lifetime of bottleneck nodes. This problem is similar to the "load balancing" problem where tasks need to be sent to one of the many servers available so that the response time is minimized – this is know to be a NP-complete problem [8].

4 Constructing Energy-Efficient Multicast Trees

In this section, we discuss algorithms for constructing energy-efficient multicast trees. As we discuss in Section 2.4, the problem of constructing energy-efficient multicast tree in a WANET is NP-complete [1, 2]. Several heuristic algorithms have been developed. In the following, we discuss several protocols for constructing energy-efficient source-based multicast trees as well as group-shared multicast trees using power control technique.

4.1 Protocols

The protocols for constructing energy-efficient multicast distribution trees can be categorized as: centralized and distributed protocols. A centralized protocol has two limitations: first, it needs global knowledge which may introduce high communication overhead especially in large scale networks; second, it would be very "expensive" if it runs repeatedly to adapt to the dynamic changes in the network, such as the remaining battery power at nodes. Since this would involve periodic sending of state information to a centralized node – a costly operation for a energy-constrained network. Given these drawbacks of a centralized approach, distributed protocols have been designed which are more suitable for the energy-efficient multicast tree problem in WANETs [14, 3, 17].

4.1.1 Centralized Protocols

Broadcast Incremental Power (BIP) algorithm is a centralized algorithm to construct a minimum-energy source-based broadcast tree [3]. BIP algorithm uses the node energy cost in Equation 3, but it neglects E^T and E^R. So BIP algorithm only considers the energy cost on the link for multicast packet transmission. BIP is similar to Prim's algorithm for constructing minimum cost spanning tree, except that it considers only "incremental energy cost" in deciding which link to add to the tree.

It constructs the tree starting from the source node, and then incrementally absorbs other nodes in the network sequentially as follows.

1. **Incremental Energy Cost Calculation:** For each node i which is already in the tree, and each node j which is not yet in the tree, the incremental energy cost associated with adding node j as node i's children is: $\Delta_{i,j} = E_{i,j} - E_i$.

2. **Absorb Node with Minimum Incremental Cost:** Find nodes i and j with the minimum value of $\Delta_{i,j}$. Absorb node j to the tree as node i's children.

As an extension of BIP, Multicast Incremental Power (MIP) algorithm is proposed for building a source-based multicast tree by eliminating all redundant transmissions (the transmissions that are not used to reach any member of the multicast group from the BIP tree) by pruning the tree [3]. To extend the network lifetime, BIP/MIP incorporates the initial battery level and residual battery level in the node cost computation [18]. To achieve reliable multicasting, Reliable BIP (RBIP) algorithm is proposed by incorporating the link error rate into the node cost for building minimum energy source-based broadcast/multicast tree [16]. It has been have proven that the approximation ratio of Minimum Spanning Tree (MST) [3] is between 6 and 12, and the approximation ratio of BIP is between $\frac{13}{3}$ and 12 [17].

Embedded Wireless Multicast Advantage (EWMA) algorithm also constructs a minimum-energy source-based broadcast tree [14]. Similar to BIP/MIP, EWMA algorithm also uses the node energy cost in Equation 3, but it neglects E^T and E^R. EWMA uses MST as the initial tree. Then, EWMA refines the initial tree to minimize TEC of the source-based broadcast tree. For example, the refinement at node i as follows.

1. **New Transmission Energy Selection:** Node i selects a downstream node, say node j. Then node i increases its transmission energy to cover all of node j's children. The incremental energy of node i is

$$\Delta E_i^j = max_{k \in j's\ children}\{E_{i,k}\} - E_i.$$

Calculate the energy $Gain$ as

$$Gain_i^j = \sum_{k \in Eliminated_i} E_k - \Delta E_i^j,$$

where $Eliminated_i$ is the set of nodes whose transmissions were eliminated when node i increased its power level.

[3] MST is built based on the link cost. So MST may not be minimum energy multicast tree in WANET.

Table 1: Characteristics of current approaches for constructing energy-efficient multicast tree in WANET.

Algorithms	Source-based	Group-shared	Min. Energy	Max. Lifetime	Reliablity	Dist./Centralized
EWMA [14]	X		X			C,D
BIP/M-IP [3, 18]	X		X	X		C
Dist-BIP-A, Dist-BIP-G [19]	X		X			D
RBIP [16]	X		X		X	C
G-REMiT [20], S-REMiT [15]	X	X	X			D
L-REMiT [21]	X			X		D

2. **Exclude Transmissions:** Selects the node j with highest positive $Gain$. Then increase transmission energy of node i to cover j, eliminate all the transmissions at node i's downstream, which are already covered by node i.

The refinement phase is from source node leaf node in EWMA.

4.1.2 Distributed Protocols

Now let us discuss some distributed algorithms for constructing energy-efficient multicast trees.

BIP algorithm has two distributed versions: Distributed-BIP-All (Dist-BIP-A) and Distributed-BIP-Gateways (Dist-BIP-G) [19]. In Dist-BIP-A algorithm, each node first constructs a BIP tree locally. Then the source node starts to broadcast its local tree structure to all of its neighbors. When a node hears from a node that already in the tree it broadcasts its own locally generated BIP tree to all of its neighbors. After several iterations, the Dist-BIP-A tree covers all of the nodes in the network. Dist-BIP-G assumes two-hop neighbors' information are available at every node. In Dist-BIP-G algorithm, source node starts to build BIP tree locally to cover its two-hop neighbors. And *gateway* node is used to denote a neighbor of the source node that connects one or more of the source node's two-hop neighbors. Only the gateway nodes of node i broadcasts the locally generated BIP tree. This reduces the number of local broadcasts leading to conservation of bandwidth and reduction in contention. Both Dist-BIP-A and Dist-BIP-G algorithms have slightly worse performance than the centralized version of the BIP algorithm.

EWMA algorithm also has a distributed version, called EWMA-Dist [14]. EWMA-Dist is organized by rounds in the refinement phase. Every node in EWMA-Dist has two-hop neighbor information. In each round, a node tries to decrease the total energy consumption (from node's point of view) by increasing its transmission power to eliminate some transmissions of its children nodes. In EWMA-Dist, the refinement starts from the source node and ends at the leaf nodes.

REMiT (Refining Energy efficiency of Multicast Trees) is a suite of distributed algorithms for enhancing energy-efficiency of multicast trees [15]. REMiT algorithms refine energy-efficiency of a pre-existing multicast tree by switching some tree nodes from one branch (forwarding) node to another. REMiT algorithms are categorized along energy-metric dimension (minimizing energy-consumption or maximizing lifetime) and multicast-tree type dimension (source-based or group-shared tree). For example, S-REMiT [15] and G-REMiT [20] are used for minimizing energy-consumption of a source-based and group-shared multicasting tree, respectively. And L-REMiT [21] can extend the tree-lifetime of a source-based multicast tree.

Now let us use S-REMiT as an example to discuss how to construct an energy-efficient source-based multicast tree in REMiT. It is divided into two phases. In the **first phase**, S-REMiT builds an initial multicast tree using available tree building algorithms, such as [22]. In the **second phase**, S-REMiT selects a node in the tree, say node i. Let x to denote the parent node of node i. Following is the S-REMiT algorithm at node i used to refine the multicast tree.

1. **New parent selection:** Node i selects a new parent node j with the highest positive energy $Gain$: $Gain_i^{x,j} := (E_i + E_x + E_j) - (E'_i + E'_x + E'_j)$, where E_i and E'_i is the energy cost of node i before and after node i changes its parent node from x to j, respectively. With the help of one-hop neighbor information, node i calculates E_i and E'_i using Equation 3, respectively.

2. **Switch to New Parent:** Node i deletes the tree link between nodes i and x and adds a tree link between nodes i and j.

The above steps is called as *a refinement* at node i. Every node in the tree does its one refinement sequentially. Every node finishes one refinement, is called *one refinement round* of S-REMiT, If there is no energy $Gain$ in a refinement round of S-REMiT, S-REMiT stops the refinement process. Then S-REMiT prunes the source-based multicast tree by deleting the redundant transmissions that are not needed to reach the members of the multicast group from the tree. After the pruning, S-REMiT will be terminated.

L-REMiT algorithm also uses refinement on the initial tree to extend lifetime of source-based multicast tree. The lifetime of a multicast tree is decided by the bottleneck nodes, which are the nodes with minimum lifetime in the tree [21]. Then L-REMiT only needs to refine the lifetime of bottleneck nodes as much as possible so that the multicast tree's lifetime is extended as follows: L-REMiT selects a bottleneck node in the tree, say node x. Let node i be the farthest (costliest) child node of node x. Following is the L-REMiT algorithm at node i used to refine the multicast tree.

1. **New parent selection:** Node i selects a new parent node j with the highest positive lifetime $LGain$: $LGain_i^{x,j} := min\{LT_i', LT_x', LT_j')\} - LT_x$, where LT_x and LT_x' is the lifetime of node x before and after node i changes its parent node from x to j, respectively.

2. **Switch to New Parent:** Node i deletes the tree link between nodes i and x and adds a tree link between nodes i and j.

After each refinement step, a new bottleneck node needs to be recomputed. The refinement is then applied to the new bottleneck node. Refinement continues till no positive lifetime gain is achieved.

Similar to S-REMiT, G-REMiT algorithm uses refinement on the initial tree to construct energy-efficient group-shared multicast tree. In a group-shared tree, when a node switches its connected tree branch to another node, energy cost of some other nodes which are even not involved in the tree branch switching may also be affected [20]. So G-REMiT is more complex in node cost computation and tree branch switching operation than S-REMiT. Table 1 shows the characteristics of the various protocols we discussed in this section.

4.2 Qualitative Analysis

In this section, we will analyze the current energy-efficient multicast tree building algorithms. For the comparison, we use propagation model $E_{i,j}^{RF} = K(r_{i,j})^{\alpha}$, where $r_{i,j}$ is the Euclidean distance between i and j, K is a constant dependent upon the properties of the antenna and α is a constant which is dependent on the propagation losses in the medium.

Let us first compare some of the current algorithms in building minimum energy source-based multicast tree. BIP/MIP algorithm constructs minimum energy multicast tree by adding nodes in the tree sequentially. So the sequential order will

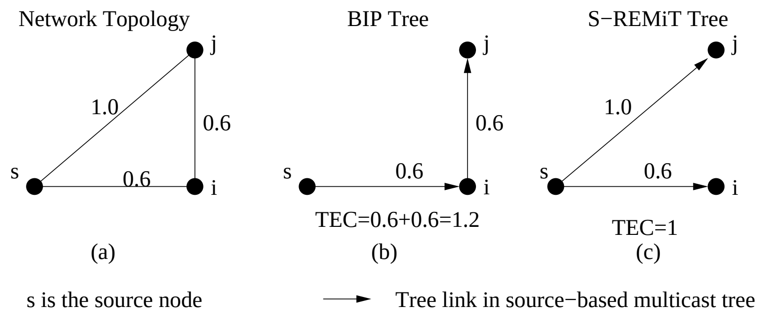

Figure 5: BIP vs S-REMiT in building source-based multicast tree. Label associated with the link is the energy cost of the link assuming that $E^T = 0$ and $E^R = 0$.

affect the performance of BIP/MIP algorithm. Also when adding a node in the tree, BIP/MIP tries to minimize the additional cost to absorb the node in the tree by minimizing the increment cost of the new link. In other words, BIP/MIP uses a link-based approach to deal with a node-based problem. Let us use an example to illustrate the pitfalls of BIP/MIP algorithm. A network topology is shown in Figure 5(a). The multicast tree generated by BIP/MIP and S-REMiT is shown in Figures 5(b) and (c), respectively. For presentation simplicity, we assume that $E^T = 0$ and $E^R = 0$ (this assumption is also used for comparison between EWMA and S-REMiT). As shown in Figure 5(b), if source node s adds nodes j first or adds nodes i and j together, then TEC of the multicast trees will be lower than if it adds node i first. So the scheme of adding nodes sequentially is not very efficient for constructing source-based energy efficient multicasting tree. EWMA algorithm is a greedy algorithm, it tries to reduce the number of downstream transmitting nodes as much as possible when there is a chance to reduce the TEC of the multicast tree. However, it neglects the contribution of nodes, whose depth are same or larger, for minimizing TEC of the tree. For example, EWMA generates a multicast tree shown in Figure 6(b). Since the depth of node j is same as node i, node j has no chance to be i's parent in EWMA algorithm. But in S-REMiT, node j will become node i's parent as shown in Figure 6(c). This is because every node has the same chance to switch its parent in S-REMiT. So S-REMiT may perform better than BIP/MIP and EWMA algorithm for building energy efficient source-based multicast tree.

Second, we discuss the current algorithms in building minimum energy group-shared multicast tree. Most of the current algorithms only try to minimize energy consumption of source-based multicast tree. A node in a source-based tree only forwards the multicast message from its parent node to its children nodes. But a

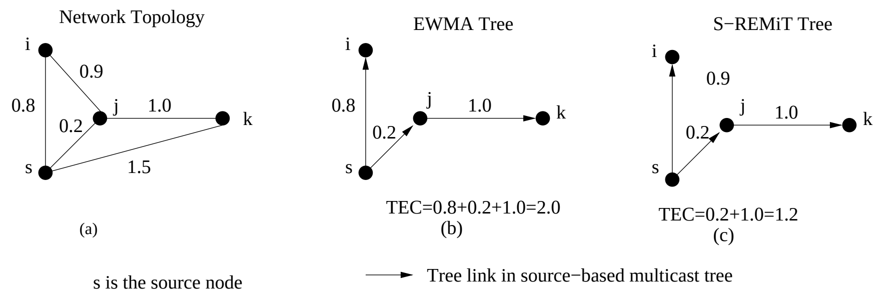

Figure 6: EWMA vs S-REMiT in building source-based multicast tree. Label associated with the link is the energy cost of the link assuming that $E^T = 0$ and $E^R = 0$.

node in a group-shared tree needs forward the multicast message to all of its connected tree neighbors except the node from which it received the multicast message. Therefore, the energy cost of the node in source-based tree is only decided by the energy cost of the links to its farthest children [14, 15]. Because the energy cost of the node in group-shared tree is not only decided by the energy cost of the links which are between the node and its connected tree neighbors but also decided by where the message is coming from. In other words, BIP/MIP and EWMA algorithms should not use energy cost of source-based multicast tree to build minimum energy group-shared multicast tree. And if a minimum energy source-based tree is used as a group-shared tree, it may not be energy efficient any more. So when the group-shared multicast tree is more energy-efficient to a specific source node, the tree will be less energy-efficient to other source nodes.

For example, in Figure 7, nine nodes are distributed in a region with dimensions 15x8. Nodes 1, 2,..., and 8 are group nodes, node 9 is not a group node, and all the group nodes have different multicast message generation rate. Node 1 is the source node when we applying MIP and EWMA algorithms. From Figure 7, we find that TEC of EWMA is highest. It means that EWMA algorithm is not energy-efficient compared with other algorithms. This is because EWMA algorithm tries to minimize the energy consumption of node 1's source-based multicast tree by minimizing the depth of the tree as much as possible. But once node 1's source-based tree is used as group-shared tree, the group-shared tree is not energy-efficient when other nodes also serve as multicast sources. However, since distributed version of EWMA (EWMA-Dist) cannot reduce the depth of the tree as much as EWMA centralized counterpart, EWMA-Dist performs better than EWMA centralized version when the tree is used as a group-shared tree. As shown in Figure 7, TEC of MIP tree is a little smaller than MST but higher than G-REMiT algorithm. It means

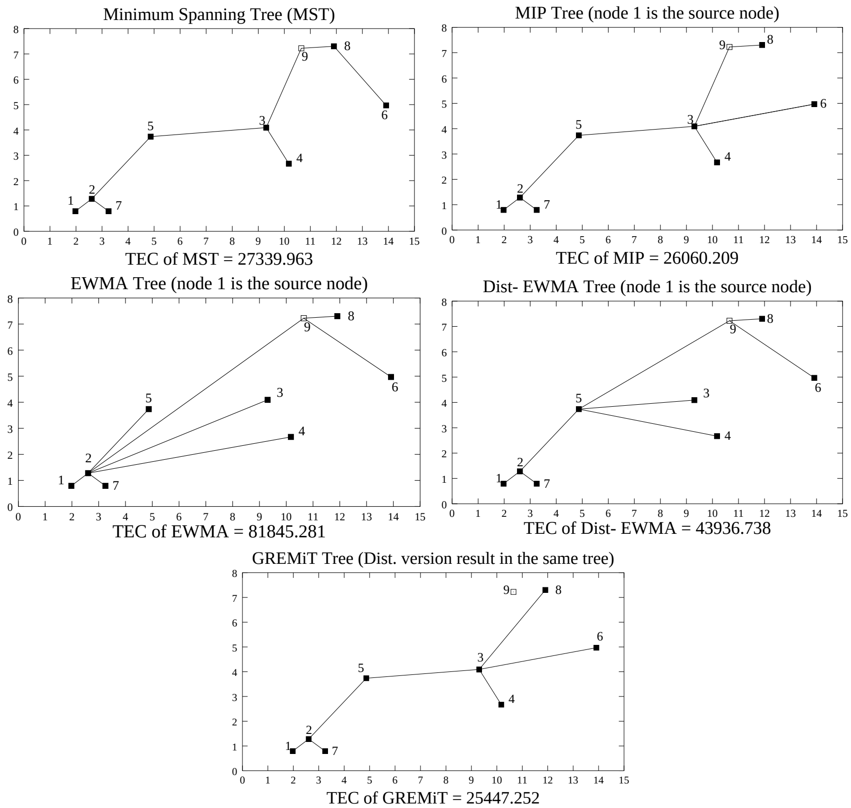

Figure 7: Different Multicast Trees generated by the algorithms with propagation loss exponent $\alpha = 2$, and $\lambda_1 = 86kb/s, \lambda_2 = 84kb/s, \lambda_3 = 80kb/s, \lambda_4 = 98kb/s, \lambda_5 = 44kb/s, \lambda_6 = 97kb/s, \lambda_7 = 13kb/s, \lambda_8 = 46kb/s$ (Node 1,2,..,8 are group nodes, but node 9 is not a group node).

that MIP algorithm performs better than MST algorithm but worse than G-REMiT algorithm.

Third, we present some results for current algorithms for optimizing lifetime of source-based multicast tree: BIP/MIP and L-REMiT. BIP/MIP uses a node cost C_i ($C_i = E_i(\frac{R_i(0)}{R_i(t)})^\beta$, where E_i is average energy cost of node i in the multicast tree for one multicast packet transmission, $R_i(0)$ and $R_i(t)$ are the residual battery energy of node i at time 0 and t, respectively, and $0 \leq \beta \leq 2$ is the weighting factor). However, this node cost C_i for BIP/MIP algorithms has following three limitations: $1)$ C_i is not node i's actual lifetime cost, even when $\beta = 1$; $2)$ Node cost is a function of time, so BIP/MIP algorithms should periodically refine the multicast tree based on the node cost. However, the BIP/MIP algorithm is not designed to refine an existing multicast tree; and $3)$ As β increases, using C_i, BIP/MIP is more likely to choose higher remaining battery energy level nodes as the forwarding nodes. But higher remaining battery energy level does not mean higher multicast lifetime. Compared with BIP/MIP, L-REMiT [21] overcomes these pitfalls. L-REMiT uses node's actual lifetime as node cost and it is refinement-based algorithm. Hence, it can be used for adapting the multicast tree to the changes of the node cost as time goes by. Currently, there is no algorithm for optimizing lifetime of a group-shared multicast tree. A possible solution is to combine G-REMiT with L-REMiT.

In the next section, we will give some of the quantitative analysis of these algorithms by simulations.

4.3 Quantitative Results

In this section, we present simulation results to compare the performance of current algorithms: BIP/MIP, EWMA, MST, REMiT. The simulations were performed using networks of four different sizes: 10, 40, 70, and 100. The distribution of the nodes in the networks are randomly generated. Every node is within the maximum transmission range (r_{max}) of at least one other node in the network. In other words, the network is connected. We use two different E^T values to represent the long range radio and short range radio. Because E^T is hardware dependent, by analyzing the experiment data in [5, 6], we decide to use $E^T = 0$ to represent long range radio and $E^T = 4Kr^{\alpha}_{max}$ to represent short range radio, where K is constant dependent upon the properties of the antenna and α is a constant which is dependent on the the propagation losses in the medium. We ran 100 simulations for each simulation setup consisting of a network of a specified size to obtain average TEC, the propagation loss exponent α is varied from 2 to 4.

Let the five types of algorithms be denoted by T_{alg}, where $alg \in A = \{$REMiT, EWMA, EWMA-Dist, MST, BIP/MIP$\}$. We use $TEC(T)$ to denote the TEC value of a tree T. We use TEC of multicast tree to define the performance metric: $Normalized\, TEC$ with algorithm alg is:

$$\frac{TEC_{alg}}{TEC_{best}},$$

where $TEC_{best} = \min(TEC(T_{alg})), alg \in A$. Using this metric, we can compare the minimum energy multicast trees obtained from REMiT, EWMA, EWMA-Dist, MST, and BIP/MIP are in terms of their TEC value.

We also use *Normalized Lifetime* as the performance metric:

$$\frac{LT(T_{alg})}{LT(T_{best})},$$

where $LT(T_{best}) = \max\{LT(T_{alg})\}, alg \in A$.

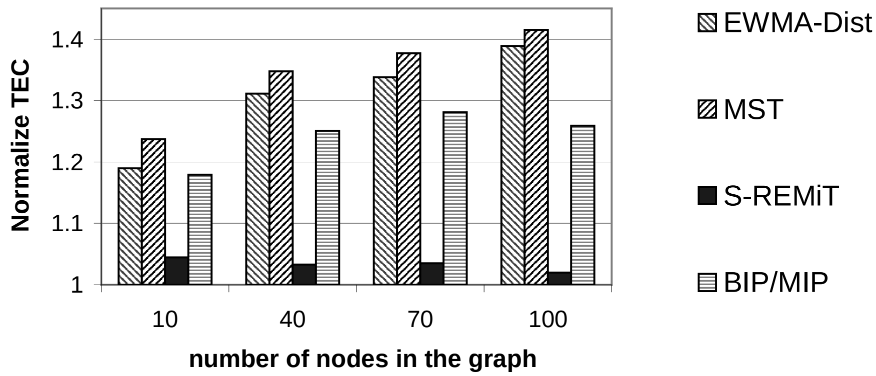

Figure 8: Average $Normalized\ TEC$ for long range radios when 50% nodes are in multicast group for source-based multicast tree ($\alpha = 2, r_{max} = 10, K = 1, E^T = 0, E^R = 0.1 * K(r_{max})^\alpha$).

Figure 8 shows the average *Normalized TEC* (shown on the vertical axis) achieved by the algorithm on networks of different size (horizon axis). The figure show that the solutions for minimum energy source-based multicast tree obtained by S-REMiT have, on the average, lower *Normalized TEC* than the other solutions (This is also true for other scenarios, such as $\alpha = 3, 4$, different group size, short range radios.).

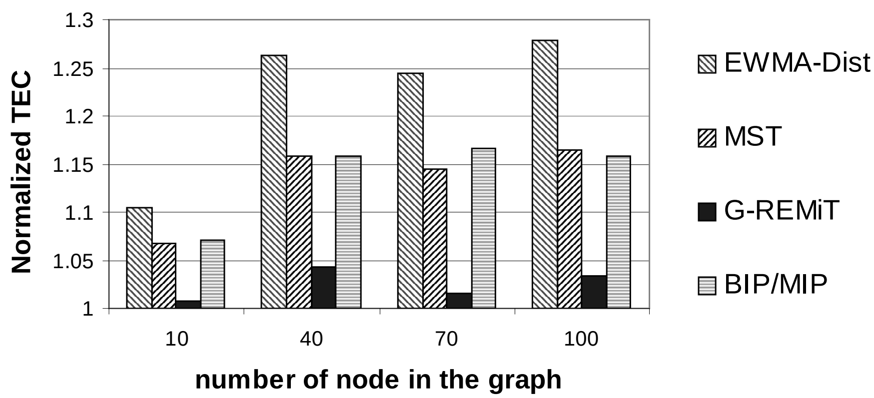

Figure 9: Average $Normalized\ TEC$ for long range radios when 100% nodes are in multicast group for group-shared multicast tree ($\alpha = 2, r_{max} = 10, K = 1, E^T = 0, E^R = 0$).

To evaluate the performance of algorithms in building group-shared multicast tree, we assume that the link bandwidth is $100kb/s$, and multicast message generation rate of every node is randomly generated as $0kb/s, 1kb/s, ..., 100kb/s$. BIP/MIP and EWMA algorithms are only used for building source-based multicast tree. In the simulations, for every group node we build a tree using BIP/MIP and EWMA algorithms with a given graph *o* and a multicast group G. We use average TEC of these trees to denote the TEC of the algorithm. For example, average TEC of MIP can be presented as average $TEC_{MIP} = \frac{1}{|G|}\sum_{\forall i \in G} TEC_{MIP}(i)$, where $TEC_{MIP}(i)$ is the TEC of node i's source-based tree generated by MIP algorithm. Similar to source-based tree comparison, in Figure 9, we can see the *Normalized TEC* achieved by these algorithms on networks of different size. Also G-REMiT has better performance than other approaches BIP/MIP, EWMA, MST algorithms for other scenarios, such as $\alpha = 3$ or 4 and different group size. But for short range radios, we found that all algorithms have very similar performance for constructing group-shared multicast tree. The main reason for such behavior is that the node's E^T is substantially greater than the maximum value of Kr_{max}^{α}. So the energy saving by transmission power control is very little. Consequently, the simplest and reasonable approach for short range radio is that every node tries to cover as many nodes as possible.

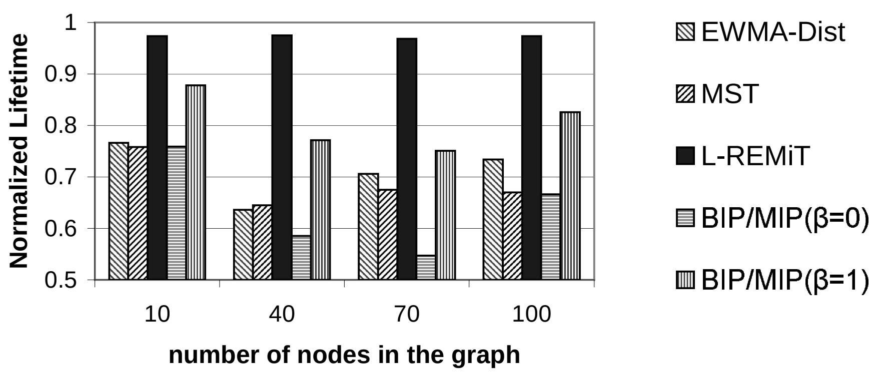

Figure 10: Average $Normalized\ Lifetime$ for long range radios when 100% nodes are in multicast group for source-based multicast tree ($\alpha = 2, r_{max} = 10, K = 1, E^T = 0, E^R = 0$).

In Figure 10, we can see L-REMiT has, on the average, higher *Normalized Lifetime* than those multicast tree obtained by other algorithms. This is also true for other scenarios, such as $\alpha = 3, 4$, different group size, and short range radios.

According to the simulations results, we find that REMiT algorithms have better performance than BIP/MIP, EWMA, and MST for various scenarios.

5 A Framework for Energy-Efficient Multicast

In this section, we present a general framework for developing energy-efficient multicast schemes, which takes into account application-specified QoS (such as reliability and delay constraints), while adapting to various network parameters (such as interference, mobility and link error rate). The framework is shown in Figure 11. Following are the components in the framework for energy efficient multicasting:

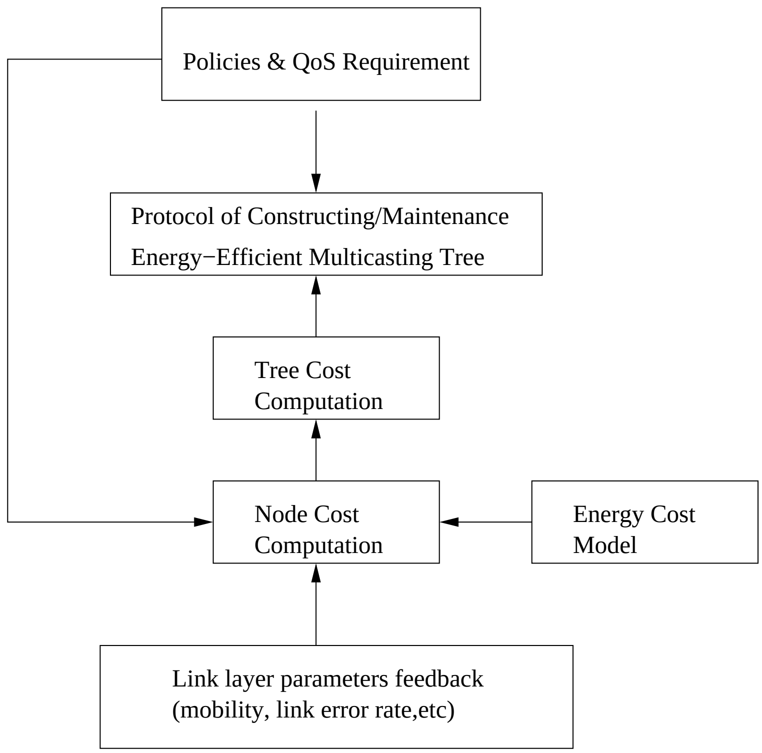

Figure 11: A Framework for Energy-Efficient Multicast.

1. **Energy Cost Model:** We have discussed energy cost model in Section 3. According to the nature of wireless transceivers, different energy cost metric can be applied.

2. **Node Cost Computation:** We have discussed this issue in Section 3.1. As shown in Figure 11, node cost computation uses application specified QoS (such as delay constraints), application specified policies (such as minimize

energy consumption and maximize lifetime), energy cost model, and wireless network parameters (such as link error rate) as inputs.

3. **Tree Cost Computation:** Using results of node cost computation, we can compute cost of the multicast tree as shown in Section 3.2.

4. **Policies and QoS Requirements:** Applications can specify optimization goals and QoS constraint (such delay and reliability) in the framework. Based on these requirements, the energy-efficient multicasting protocol can choose different scheme to build energy-efficient multicast tree.

5. **Link layer parameters feedback:** To adapt to the dynamic features of wireless network, several parameters of wireless link need to be sensed, such as the link error rate. The link error rate can be determined by maintaining the history of retransmission on a wireless link. If a packet is transmitted n times in average on a wireless link to be received successfully, we would calculate the link error rate of the link as $1 - \frac{1}{n}$. For example, if $n = 4$ then the link error rate is $1 - \frac{1}{4} = 0.75$.

6. **Protocol for energy-efficient multicasting:** The protocol includes several different multicast tree constructing and maintenance/refinement algorithms. We discuss these algorithms in Section 4.

This framework also involves cross layer design. It needs to combine network layer and link layer together. Node cost computation, tree cost computation and the protocol are located at network layer, whereas network layer computes the node cost and uses the general energy-efficient protocol to reduce energy consumption or extend the lifetime of multicast tree. The link layer uses link layer parameters as the feedbacks to network layer.

6 Conclusions

In this chapter, we presented general techniques for conserving energy in wireless communication. Then we described the metrics for modeling cost of energy consumption and different optimization goals for energy-efficient multicasting. We also discussed the existing solutions for constructing energy-efficient multicast distribution trees for WANETs and gave the qualitative analysis and simulations results of these solutions. Finally, we presented a framework for energy-efficient multicasting.

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