BER Performance Analysis of an On-off Keying based Minimum Energy Coding for Energy Constrained Wireless Sensor Application

Qinghui Tang
Dept. of Electrical Engineering
qinghui.tang@asu.edu
Arizona State University
Tempe, AZ 85287

Sandeep K. S. Gupta
Dept. of Computer Science and Engineering
sandeep.gupta@asu.edu
Arizona State University
Tempe, AZ 85287

Loren Schwiebert
Department of Computer Science
loren@wayne.edu
Wayne State University
Detroit, MI 48202

***Abstract*— An On-Off keying based minimum-energy coding scheme with coherent receiver has been shown to provide better performance than BPSK. This paper presents a closed-form expression of the BER performance of that scheme over an AWGN channel with either a coherent receiver or a noncoherent receiver. In this paper, a more conservative result is obtained due to a better characterization of the average energy per source bit. Our results show that the performance is better than BPSK only when the codeword length is greater than 63. It is shown that hard-decision decoding outperforms BPSK only when the SNR is higher than a certain threshold and that soft-decision decoding outperforms BPSK regardless of the SNR value. Recommendations for practical codeword lengths are also provided.**

I. Introduction

Many energy constrained applications demand an energy-efficient design to minimize power consumption and to extend operation lifetime. Having the underlying transmission and communication mechanism energy-efficient is critical to improve the needed efficiency. Especially given the energy used to transmit a single bit equals the energy needed to perform a thousand 32-bit calculations [1].

In previous work [2], we proposed a Minimum Energy coding (ME-coding) based on On-Off Keying (OOK) . The basic idea is to map source symbols into codewords with fewer high bits, hence transmitting as few high bits as possible in order to save transmission energy. For energy-constrained and low data rate applications, such as medical monitoring and prosthesis [3], we can afford to sacrifice bandwidth efficiency to achieve energy efficiency as the total data rate is in the order of a few kilobytes per second. Further, the fact that ME-coding uses OOK, which is characterized by the simplicity of the circuitry, makes it an excellent solution for system miniaturization. In this work, we derive closed-form expressions of ME-coding over Additive White Gaussian Noise (AWGN) channels. For simplicity purposes, a coherent receiver is rarely used in conjunction with OOK due to the complex signal and local carrier. Hence, we extend previous work [2] by incorporating noncoherent decoding in the analysis. Further, we realized that [2] miscalculated the average energy per bit which led to overoptimistic results. We address the problem herein.

We compare the performance of ME-coding mainly with Binary Phase Shift Keying (BPSK) because the simplicity and energy efficiency of BPSK [4] are preferred for low data rate and energy constrained applications. Our analysis and simulation show that although the performance of ME-coding is not as good as the previous work [2] claims, ME-coding still outperforms BPSK when its codeword length is greater than 63 and the Signal-to-Noise Ratio (SNR) is relatively high. We also show that hard-decision decoding outperforms BPSK only when the SNR is higher than a certain threshold and that soft-decision decoding outperforms BPSK regardless of the SNR value. We discuss the complexity of ME-coding and suggest that a practical codeword length should be in the range from 63 to 1023, according to the performance requirement of the clock recovery circuit of the receiver.

The rest of the paper is organized as follows. In Section II we review related work on minimum energy coding and the preliminaries used in our analysis. Performance analysis and comparison of hard-decision decoding and soft-decision decoding are developed in Sections III and IV, respectively. We discuss the complexity and the feasible range of codeword length and conclude in Section V.

II. Preliminaries

A. Related Work

Wang *et al.* [4] showed BPSK is a preferred energy efficient scheme for low-data rate applications. A simple digital modulation technique such as OOK is totally ignored due to its low error performance. ME-coding was originally proposed by Erin and Asada [5]. In an effort to take advantage of OOK's feature (i.e., having to transmit only high bits), they map the source data bits into codewords which have fewer high bits in them and codewords with fewer high bits are assigned to messages with higher probabilities.

In [2], a new ME-coding scheme is proposed. As shown in Fig. 1, the basic idea involves mapping every k bits of a source bitstream into an n-bit ($n = 2^k - 1$) codeword. The all-zeros source symbol S_0 is mapped into a n-bit all-zeros codeword M_0. All other source symbols S_i are mapped into n-bit codewords M_i with only one high bit (namely, the i^{th} bit in the codeword) to reduce the transmission energy consumption. It seems the average energy consumed by ME-coding to transmit ME codeword is $\frac{1}{n}$ of BPSK. However, previous

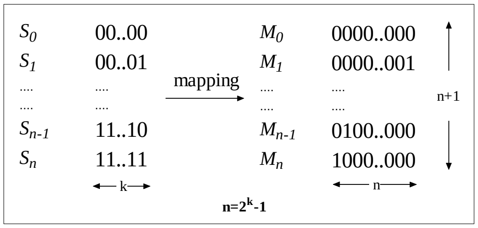


Fig. 1. ME encode, transmission and decode.

work [2] failed to consider that with an n-bit codeword, BPSK can transmit n bits of source data, whereas ME-coding can transmit k bits of source data. Therefore, the average energy consumed per source bit of ME-coding is $\frac{1}{k}$ of BPSK as opposed to $\frac{1}{n}$.

B. Coherent Receiver vs. Noncoherent Receiver

On an AWGN channel, for an ideal coherent receiver, when 1 or 0 is transmitted, the distributions of the output signal r of the detector are

$$p_1(r) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(r-A)^2}{2\sigma^2}\right) \tag{1a}$$

and

$$p_0(r) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-r^2}{2\sigma^2}\right), \tag{1b}$$

respectively, where A is the amplitude of modulated signal and σ is the standard deviation of additive white noise. For a noncoherent receiver, the distributions of the output signal of the detector are [6]

$$p_1(r) = \frac{r}{\sigma^2} exp\left(-\frac{r^2+A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right) \tag{2a}$$

and

$$p_0(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right), \tag{2b}$$

respectively, where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind. Denote P_{eS} and P_{eM} as the error probability of a high bit being received as a low bit and a low bit being received as a high bit, respectively, then for a coherent receiver we have [7]

$$P_{eS} = P_{eM} = \frac{1}{2}\,\mathrm{erfc}(\sqrt{E_b/4N_0}), \tag{3}$$

where $\frac{E_b}{N_0}$ is the energy-to-noise spectral density ratio and $\mathrm{erfc}(\cdot)$ is the complementary error function. For a noncoherent receiver we have [6]

$$P_{eM} = 1 - Q\left(\sqrt{2E_b/N_0}, \frac{b}{\sigma}\right) \text{ and } P_{eS} = \exp\left(-\frac{b^2}{2\sigma^2}\right), \tag{4a}$$

where b is the decision threshold defined [6] as

$$b = \sigma\sqrt{2 + E_b/2N_0}, \tag{5}$$

and $Q(\cdot)$ is the Marcum-Q function. Then we can calculate the theoretical BER of OOK for both a coherent receiver and a noncoherent receiver as

$$P_e = \frac{1}{2}[P_{eS} + P_{eM}]. \tag{6}$$

C. Number of erroneous bits and illegal codewords

When an ME codeword M_i is wrongly received as M_j and decoded as source symbol S_j, not all k bits differ from the original source bits. For example, when $k = 3$, source symbol (0 1 1) is mapped into ME codeword (0 0 0 0 1 0 0), and wrongly received as ME codeword (1 0 0 0 0 0 0). Then the decoded symbol at the receiver is (1 1 1); there is only one bit error when compared against (0 1 1). The actual number of bit errors is the Hamming distance between symbol S_i and symbol S_j, denoted as d_{ij}. It is easy to get

$$\sum\nolimits_{j=0}^{n} d_{ij} = k \cdot 2^{k-1}, \tag{7}$$

which will be used later in derivations of a closed-form expression. If we receive an illegal codeword, a codeword with more than one high bit, we do not consider all the original k bits in error. Instead, we can permanently decode it as a source symbol, e.g., S_j. When receiving an illegal codeword, the average bit error rate upon receiving the illegal codeword is

$$P_{mbe} = \sum\nolimits_{i=0}^{n} P\left\{S_i|\text{illegal codeword}\right\} \cdot \frac{d_{ij}}{k}. \tag{8}$$

Assuming the distribution of source symbols is uniform and considering (7), we have

$$P_{mbe} = \frac{1}{n+1}\sum\nolimits_{i=0}^{n} \frac{d_{ij}}{k} = \frac{1}{2}. \tag{9}$$

So we can permanently decode an illegal codeword to some fixed symbol, and the average bit error rate is $\frac{1}{2}$. Thus we can improve the performance a little compared with classifying all k bits as wrong bits ($P_{mbe} = 1$).

III. Hard-decision decoding

The traditional way to receive and decode ME codeword is hard-decision. There are two different situations, original symbol is not all-zero and original symbol is all-zero.

A. Original symbol is not all-zero

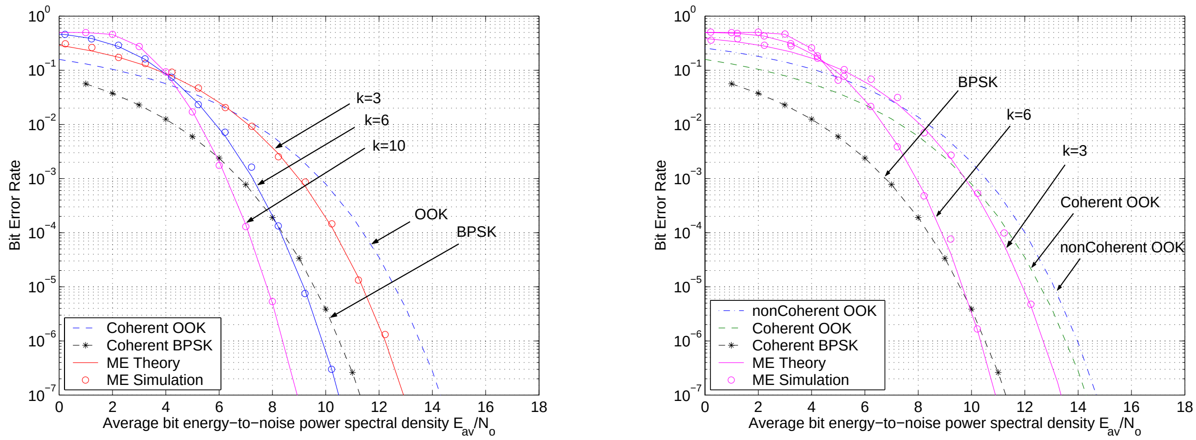
When the original k-bit source symbol S_i is not all-zero ($i \neq 0$), the encoded ME codeword M_i can be received as M_i, or received as another codeword M_j (original high bit changes to low bit, and another low bit received as high bit), or received as other illegal codewords. We summarize the related pairwise receiving probability and bit error rate in Table I. Then the average bit error rate when transmitting S_i is

$$P_e\{S_i\} = P_{m0}\frac{d_{i0}}{k} + \sum\nolimits_{j\neq i} P_{ij}\frac{d_{ij}}{k} + P_{mb}P_{mbe}. \tag{10}$$

B. Original symbol is all-zero

Similarly, when the original source symbol is composed of only zeros, the symbol S_0 is encoded as ME codeword M_0. We have the pairwise receiving probability and bit error rate as in Table I. Then the average bit error rate when transmitting S_0 is

$$P_e\{S_0\} = P_{eS}(1-P_{eS})^{n-1}\sum\nolimits_{j=1}^{n} \frac{d_{0j}}{k} + P_{mb}P_{mbe}. \tag{11}$$



(a) Coherent receiver.

(b) Noncoherent receiver.

Fig. 2. Bit Error Rate vs. average energy-to-noise ratio with hard-decision decoding

TABLE I

PAIRWISE ERROR PROBABILITY OF HARD-DECISION DECODING.

Not all zero symbol

M_i Received & Decoded as	Probability	BER
$M_0 \rightarrow S_0$	$P_{m0} = P_{eM}(1-P_{eS})^{n-1}$	$\frac{d_{i0}}{k}$
$M_i \rightarrow S_i$	$P_{mc} = (1-P_{eM})\,(1-P_{eS})^{n-1}$	0
$M_j \rightarrow S_j$, $j \in [1\ n]$, $j \neq i, 0$	$P_{ij} = P_{eM}P_{eS}(1-P_{eS})^{n-2}$	$\frac{d_{ij}}{k}$
Illegal codeword	$P_{mb} = 1 - P_{m0} - P_{mc} - \sum_{j \neq i,0} P_{ij}$	P_{mbe}

All zero symbol

M_0 Received & Decoded as	Probability	BER
$M_0 \rightarrow S_0$	$P_{m0} = (1-P_{eS})^n$	0
$M_j \rightarrow S_j$, $j \in [1\ n]$, $j \neq i, 0$	$P_{0j} = P_{eS}(1-P_{eS})^{n-1}$	$\frac{d_{0j}}{k}$
Illegal codeword	$P_{mb} = 1 - P_{m0} - \sum_{j \neq 0} P_{0j}$	P_{mbe}

With (10), (11), and the assumption of uniform distribution of source symbols, we have

$$
\begin{aligned}
P_e\{all\} &= \frac{1}{n+1}\left[\sum\nolimits_{i=1}^{n} P_e\{S_i\} + P_e\{S_0\}\right] \qquad (12)\\
&= \frac{1}{n+1}[\frac{n+1}{2}P_{eM} + \frac{n+3}{2}P_{eS} - P_{eM}P_{eS} \\
&- P_{eS}^2 - \frac{1}{2}(n+1)](1-P_{eS})^{n-2} + \frac{1}{2}.
\end{aligned}
$$

C. Numerical and Simulation Results

Define E_b as the energy used to send one modulation symbol and E_{av} as the average energy used to transmit a source data bit [6]. For BPSK, E_{av} equals E_b, for OOK, E_{av} equals $\frac{E_b}{2}$. For ME-coding, one k-bit non-zero source symbol is transmitted as one n-bit ME codeword which has only one high bit; transmitting an all-zero source symbol does not consume energy. The average energy consumed per source bit is $\frac{E_b}{k}\frac{2^k-1}{2^k}$. For large values of k, we can simply approximate it as $\frac{E_b}{k}$.

Fig. 2(a) show the average bit energy-to-noise ratio of ME-coding compared against BPSK and coherent OOK with various codeword lengths. It also shows the results of Monte Carlo simulation for different k, which validates the curves of the closed-form expressions. When $k = 3$, ME-coding has performance similar to coherent OOK, but worse than BPSK. For $k = 6$ and $k = 10$, ME-coding has worse performance in low $\frac{E_{av}}{N_o}$, but has better performance when $\frac{E_{av}}{N_o}$ is higher than $8dB$ ($k = 6$) or $6dB$ ($k = 10$). This indicates that for a large codeword length, ME-coding still can either achieve energy efficiency under the same BER or achieve better performance under the same energy consumption.

Fig. 2(b) shows the performance of ME-coding with a noncoherent receiver. Compared with Fig. 2(a), we observe the performance of noncoherent ME-coding is slightly degraded from coherent ME-coding of about $0.25dB$. The benefit of using noncoherent ME-coding is its simplicity which can compensate for the small performance deterioration.

IV. SOFT-DECISION DECODING

Since ME-coding with hard-decision decoding does not have any error recovery capability, the conception of soft-decision decoding (code-by-code detection) was introduced in [2] to improve the performance of ME-coding. Instead of bit by bit sampling and comparison, the signal amplitudes of all the n bits of the same codeword are compared with each other; the bit with the highest strength is decoded as a "one" and the rest are decoded as "zero". Fig. 3 illustrates the process of soft-decision decoding: with hard-decision decoding, it would have been detected as (0 1 1 1 0 0 1); if we look at the bit with highest strength, the entire codeword is detected as (0 0 1 0 0 0 0). In this way we can form a legal codeword and decode it, eliminating the occurrence of an invalid codeword at the receiver.

A. Original symbol is not all-zero

Similar to the hard-decision decoding in Section III, when the original k-bit source symbol is not equal to zero, we can

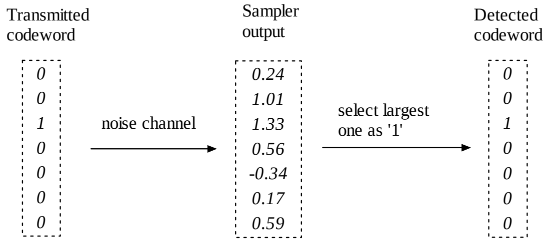


Fig. 3. Illustration of soft-decision decoding.

summarize the related pairwise probability and bit error rate as in Table II.

TABLE II
PAIRWISE ERROR PROBABILITY OF SOFT-DECISION DECODING.

Not all zero symbol		
M_i Received & Decoded as	Probability	BER
$M_0 \rightarrow S_0,$	$P_{m0} = P_{eM}(1-P_{eS})^{n-1}$	$\frac{d_{i0}}{k}$
$M_i \rightarrow S_i,$	P_{mc}	0
$M_j \rightarrow S_j,\ j \in [1\ n],\ j \neq i$	P_{ij}	$\frac{d_{ij}}{k}$
All zero symbol		
M_i Received & Decoded as	Probability	BER
$M_0 \rightarrow S_0,$	$P_{m0} = (1-P_{eS})^n$	0
$M_j \rightarrow S_j,\ j \in [1\ n]$	P_{0j}	$\frac{d_{j0}}{k}$

P_{mc} is defined as the probability of correctly receiving a ME codeword, or the probability that the amplitude of the original high bit is still larger than all the other $n-1$ bits. So the contaminated original high bit (denoted as y) has to be in the range from b to ∞ to be judged as one, and all other bits must be less than y. With (1a) and (1b) we have

$$P_{mc}=\int_b^{\infty}\left[\int_{-\infty}^{y}\frac{1}{\sqrt{2\pi}\sigma}exp[\frac{-r^2}{2\sigma^2}]dr\right]^{n-1} \frac{1}{\sqrt{2\pi}\sigma}exp[\frac{-(y-A)^2}{2\sigma^2}]dy. \quad (13)$$

By (2a) and (2b) we have P_{mc} for the noncoherent receiver

$$P_{mc}=\int_b^{\infty}\left[\int_0^{y}\frac{r}{\sigma^2}exp\left(-\frac{r^2}{2\sigma^2}\right)dr\right]^{n-1} \frac{y}{\sigma^2}exp\left(-\frac{y^2+A^2}{2\sigma^2}\right)I_0\left(\frac{Ay}{\sigma^2}\right)dy. \quad (14)$$

P_{ij} is the probability codeword M_i is received as M_j, i.e., the probability that the original high bit becomes smaller than the threshold and another low bit is detected as "high" because the additive noise makes it the strongest one and larger than the threshold (amplitude x ranges from the threshold b to ∞; the other $n-2$ bits and the original high bit are less than x). For the coherent receiver, P_{ij} is given by

$$P_{ij}=\int_b^{\infty}\frac{1}{\sqrt{2\pi}\sigma}exp\left(\frac{-x^2}{2\sigma^2}\right)\left[\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}\sigma}exp[\frac{-r^2}{2\sigma^2}]dr\right]^{n-2} \left[\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}\sigma}exp\left(\frac{-(r-A)^2}{2\sigma^2}\right)dr\right]dx, \quad (15)$$

and for the noncoherent receiver it is

$$P_{ij}=\int_b^{\infty}\frac{x}{\sigma^2}exp\left(-\frac{x^2}{2\sigma^2}\right)\left[\int_0^{x}\frac{r}{\sigma^2}exp\left(-\frac{r^2}{2\sigma^2}\right)dr\right]^{n-2} \left[\int_0^{x}\frac{r}{\sigma^2}exp\left(-\frac{r^2+A^2}{2\sigma^2}\right)I_0\left(\frac{Ar}{\sigma^2}\right)dr\right]dx. \quad (16)$$

We can numerically calculate P_{mc} and P_{ij}. Then the average bit error rate when transmitting S_i is:

$$P_e\{S_i\}=P_{m0}\frac{d_{i0}}{k}+\sum_{j\neq i, j\neq 0}P_{ij}\frac{d_{ij}}{k}. \quad (17)$$

B. Original symbol is all-zero

When the original source symbol is composed of only zeros, the symbol S_0 is mapped as ME codeword M_0. We have the pairwise probability and bit error rate as in Table II.

The probability M_0 is received as M_j occurs only when the j^{th} bit changes to a high bit above the threshold and all the other $n-1$ bits, so for the coherent receiver we have

$$P_{0j}=\int_b^{\infty}\left[\int_{-\infty}^{y}\frac{1}{\sqrt{2\pi}\sigma}exp[\frac{-r^2}{2\sigma^2}]dr\right]^{n-1}\frac{1}{\sqrt{2\pi}\sigma}exp[\frac{-y^2}{2\sigma^2}]dy. \quad (18)$$

For the noncoherent receiver we have

$$P_{0j}=\int_b^{\infty}\left[\int_0^{y}\frac{r}{\sigma^2}exp\left(-\frac{r^2}{2\sigma^2}\right)dr\right]^{n-1}\frac{y}{\sigma^2}exp\left(-\frac{y^2}{2\sigma^2}\right)dy. \quad (19)$$

Then the average bit error rate when transmitting S_0 is:

$$P_e\{S_0\}=\sum_{j=1}^{n}P_{0j}\cdot\frac{d_{j0}}{k}. \quad (20)$$

From (17) and (20), we can get the total bit error rate for soft-decision decoding as

$$P_e\{all\}=\frac{1}{2}P_{m0}+\frac{n}{2}P_{ij}+\frac{1}{2}P_{0j}. \quad (21)$$

C. Numerical and Simulation Results

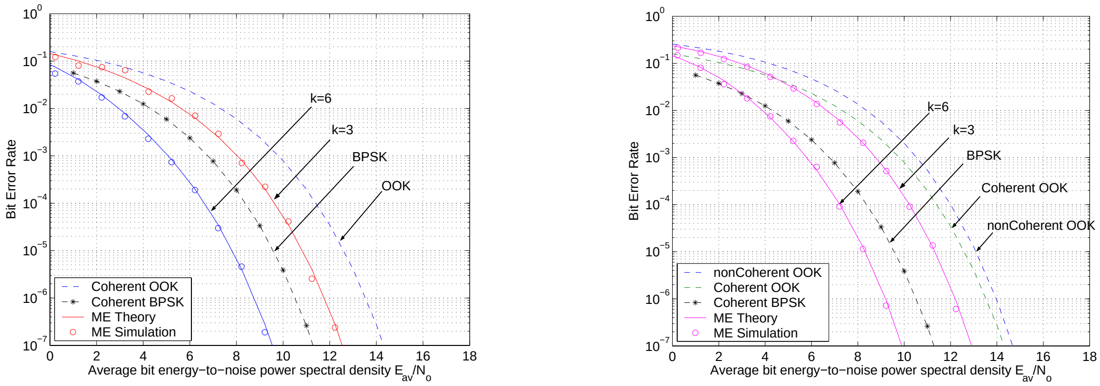
In Fig. 4(a), the theoretical value and Monte Carlo simulation results are illustrated. ME-coding with soft-decision decoding has consistently better performance than coherent BPSK when $k=6$. When BER is 10^{-7} and k is 6, coherent ME-coding shows a $1.7dB$ gain over BPSK, which means an energy savings of about 32.5% ($10log(\frac{1}{0.675})\approx 1.7dB$). Again, Fig. 4(b) shows that with soft-decision decoding the performance of the noncoherent receiver is slightly worse than the coherent receiver. But a gain of $1.4dB$ at BER 10^{-7} can still guarantee an energy saving about 27.6% ($10log(\frac{1}{0.724})\approx 1.4dB$).

Comparing Fig. 4(a) with Fig. 2(a), we observe the performance of soft-decision decoding improves about $0.9dB$ over hard-decision decoding when $BER=10^{-7}$.

V. DISCUSSION AND CONCLUSION

A. Complexity vs performance

Clearly, coherent soft-decision decoding has the best result but it needs to hold all n amplitude values and compare them, which will increase the circuit complexity and die area of



(a) Coherent receiver. (b) Noncoherent receiver.

Fig. 4. Bit Error Rate vs. average energy-to-noise ratio with soft-decision decoding

the receiver. Whereas noncoherent hard-decision decoding has the lowest performance and simplest design. Hard-decision decoding does not have the consistent performance of soft-decision decoding, thus it can be used in a relatively stable channel, where signal noise ratio generally remains at a high level. Soft-decision decoding can be used in a relatively protean channel, where signal noise ratio fluctuates in a large range, and the performance consistently outperforms that of BPSK at most times. We draw approximate comparisons of the four schemes in Fig. 5 in terms of performance and complexity. We should recognize, as k increases, the resulting complexity and performance of soft-decision decoding will increase. At a certain point, the increased complexity and the increased circuit power consumption of noncoherent soft-decision decoding may exceed that of coherent hard-decision decoding. Exactly when this would happen depends on the detailed implementation of the circuit and the value of k.

B. Codeword Length

ME-coding will increase the bandwidth requirement by $\frac{n}{k}$ times compared with BPSK. For energy-constrained low data rate applications, we can safely sacrifice bandwidth efficiency to obtain energy efficiency. For example, $k = 6$ indicates a ME codeword length of 63. Considering several all-zero symbols may be transmitted consecutively, then the possible length of a long string of zeros may be several multiples of 63. A large k value may require a high performance synchronization design which will complicate the design of the receiver. For instance, the T1/E1 line interface chip LXT336 [8] is capable of detecting 175 continuous zeros before it sends out a *Loss of Signal* error. A high-speed SDH chip [9] may tolerate as many as 2000 consecutive zeros. Since we can obtain performance and energy efficiency only if $k \geqslant 6$, we suggest a reasonable and practical range for k is from 6 to 10 (n from 63 to 1023). A longer codeword length is possible, but will require higher capability of the clock recovery.

C. Conclusion

Compared with coherent BPSK, ME-coding with a large codeword length can achieve either performance improvement or energy efficiency. Its energy efficiency and circuit simplicity

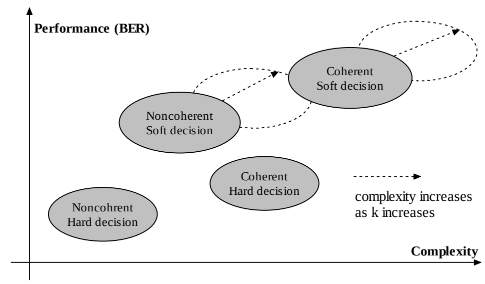


Fig. 5. Performance vs. Complexity

make ME-coding an excellent option for energy constrained, resource limited wireless sensor applications. Our result also shows hard-decision decoding can be used only under relatively large SNR, and soft-decision decoding does not have this constraint. Regarding complexity and performance, choices need be made carefully to choose between hard-decision or soft-decision decoding, coherent receiver or noncoherent receiver.

REFERENCES

[1] K. Barr and K. Asanovic, "Energy aware lossless data compression," in *Proceedings of the 1th International Conference on Mobile Systems, Applications, and Services MobiSys 2003*, May 2003. [Online]. Available: http://www.usenix.org/events/mobisys03/tech/barr.html

[2] Y. Prakash and S. K. S. Gupta, "Energy efficient source coding and modulation for wireless applications," in *Proc. IEEE WCNC'03*, vol. 1, New Orleans, LA, USA, Mar. 2003, pp. 212–217.

[3] L. Schwiebert, S. K. S. Gupta, P. S. G. Auner, G. Abrams, R. Lezzi, and P. McAllister, "A biomedical smart sensor for the visually impaired," in *IEEE Sensors 2002*, Orlando, FL., USA, June 2002.

[4] A. Wang, S. Cho, C. Sodini, and A. Chandrakasan, "Energy efficient modulation and MAC for asymmetric RF microsensor systems," in *Proceedings of the 2001 international symposium on Low power electronics and design*. ACM Press, 2001, pp. 106–111.

[5] A. C. Erin and H. H. Asada, "Energy optimal codes for wireless communications," in *Proc. IEEE 38th Conf. Decision and Control (CDC03)*, Phoenix, AZ, USA, Dec. 1999.

[6] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. IEEE Press., 1996.

[7] J. G. Proakis, *Digital Communications*, 4th ed. New York, NY, USA: McGraw-Hill, 2001.

[8] "LXT336 Quad T1/E1 Receiver," Intel, 2001.

[9] "MAX3881 2.488Gbps SDH/SONET deserializer with clock recovery," MAXIAM, 2001.