Localization Using Signal Strength: To Range Or Not To Range?

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ABSTRACT

Received Signal Strength (RSS) data collected within a wireless network can be used to obtain either range estimates or connectivity information. Both approaches lead to localization schemes that require no additional hardware. It is not clear, however, when a range-based scheme should be used in favor of a connectivity-based one. We use analysis of the Fisher information and the Cramér–Rao Bound (CRB) to characterize the error of both approaches. We find the existence of a *critical connectivity* value, below which the use of RSS data for range-based localization is counter-productive. We show that an approximation of the critical connectivity value can be computed as a function of the network size and the parameters of the propagation model.

Categories and Subject Descriptors

C.2 [**Computer Communication Networks**]: Network Protocols.

General Terms

Algorithms, Performance, Theory.

Keywords

Localization, Signal Strength, RSS, Ranging, Connectivity, Fisher Information, Cramér–Rao Bound, Approximation.

1. INTRODUCTION

The *Received Signal Strength* (RSS) values measured by most radio transceivers can be used to estimate the distance between nodes and implement *range-based* localization schemes (e.g. [11]). These schemes are popular because no additional hardware is required on the nodes to localize. Their accuracy, however, is often questioned. Given the variability of the wireless channel, range estimates using RSS are inaccurate by nature and can lead to large localization errors. For this reason, several authors have proposed *range-free* schemes that implicitly apply binary quantization to the RSS data, and localize the nodes using only the connectivity information (e.g. [6, 12]).

Should the RSS data be used for range estimates, or should they be converted into connectivity information? Which approach works better? We answer these questions by using a parameter estimation approach and by comparing the theoretical limits that bound the error in the two cases [8, 9]. Using this framework, theoretical tools such as the Fisher information and the Cramér–Rao Bound (CRB) allow us to investigate the problem on a general basis, without limiting our analysis to any particular scheme. The aim of our work is to provide a practical rule to help system designers to identify the conditions under which a localization approach works better than the other. Despite the attention received by RF-based localization schemes, to our knowledge, this is the first time that this problem has been addressed.

In Section 2 we use a simple localization example to introduce the Fisher information for RSS and connectivity data when the signal strength follows the log-normal shadowing model. In Section 3, we use the CRB analysis to characterize the error when localization involves multiple nodes deployed as a network. By comparing the CRB for the two cases, we show the existence of a *critical connectivity* ($c_{\rm cr}$) value. For a network with connectivity below $c_{\rm cr}$, the minimum error achievable by a range-free scheme is potentially lower than the error of one that uses RSS range estimates. The opposite is true for network with connectivity greater than $c_{\rm cr}$.

Knowledge of the $c_{\rm cr}$ value defines the choice of the approach to use. However, computing this value requires knowing the true node positions. After studying the properties of the $c_{\rm cr}$ value, in Section 3.4 we investigate how to approximate it. Using extensive simulations we show that the $c_{\rm cr}$ value can be approximated with sufficient accuracy by using a function of the network size and the parameters of the propagation model. We finally present a test case where the approximation found is used to choose between a range-based and a range-free scheme; analysis of the localization error supports the choice made by using our results.

2. 1D NODE LOCALIZATION

Suppose two devices are placed as in Figure 1, and we want to compute the position of node 1, or equivalently its distance d from the origin. We will estimate d using two solutions: one that uses RSS value collected between the two nodes, and another that converts them into connectivity data. After describing the propagation model for the RF signal, we will compare the two approaches.

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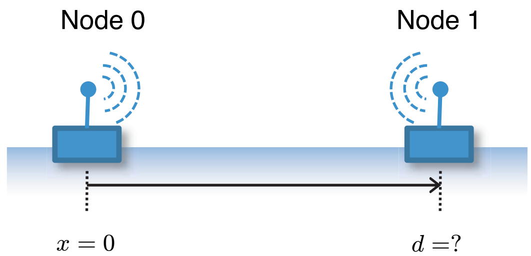


Figure 1: The distance of node 1 from the origin has to be estimated using radio messages.

2.1 Propagation Model

We assume that the RSS values follow the *log-normal shadowing* model, a propagation model widely used in link budget analysis [10]. Let $\{z_1, z_2, z_3, \ldots\}$ be the set of RSS measurements collected between the two nodes, and let p_{01} be the average[1] of such values. If the RSS is measured in dB or dBm, then p_{01} is the outcome of a random variable P_{01} with normal distribution:

$$\begin{aligned} P_{01} &\sim \mathcal{N}(\bar{P}_r(d), \sigma_{\text{dB}}) \\ \bar{P}_r(d) &= P_0 - 10\, n_{\text{p}} \log_{10}(d/d_0). \end{aligned} \tag{1}$$

where P_0 is the received power measured at reference distance d_0, and n_{p} is the *path loss exponent*. The standard deviation σ_{dB} models the variability measured between pairs of nodes with the same separation distance, but placed in different locations.

2.2 Range-Based Localization

The first solution is to estimate d using the average RSS measured between the nodes. Having assumed the model (1), we can estimate d using, for example, the *Maximum Likelihood Estimator* (MLE):

$$\hat{d}_{\text{ML}} = d_0 10^{(P_0 - p_{01})/10 n_p}. \tag{2}$$

Assuming that the path loss exponent n_{p} is known, the MLE provides a simple solution to convert RSS values into range estimates. Using (2), we can also evaluate the estimation error. If the measurement is $p_{01}{=}\bar{P}_r(d){+}\ \delta$, where δ is a sample from the random variable $\Delta \sim \mathcal{N}(0, \sigma_{\text{dB}})$, then the error is:

$$e = \hat{d}_{\text{ML}} - d = d\left(10^{\frac{-\delta}{10 n_p}} - 1\right). \tag{3}$$

In absence of shadowing effects ($\delta = 0$), the MLE produces the correct estimates (i.e. $e = 0$). When $\delta \neq 0$, the error is proportional to the distance between the nodes; therefore, range estimates for nodes with a large separation distance are less accurate than range estimates for nodes that are close to each other.

2.3 Connectivity-Based Localization

The second option is to estimate d using connectivity data. Using the approach proposed by Patwari and Hero III [9], we obtain connectivity measurements by comparing the average RSS against a threshold P_{th}. The two nodes are "connected" if $p_{01} \geq P_{\text{th}}$ or "disconnected" in the other case. According to this binary quantization, the connectivity is described by a random variable C_{01}:

$$C_{01} = \begin{cases} 0 & \text{if } p_{01} < P_{\text{th}} \quad \text{(nodes disconnected)} \\ 1 & \text{if } p_{01} \geq P_{\text{th}} \quad \text{(nodes connected).} \end{cases} \tag{4}$$

A connectivity-based scheme will use the intuitive assumption that two nodes are "close" if $C_{01} = 1$, and "far" if $C_{01} = 0$. The advantage of this approach is that the position can be estimated without knowing the propagation model's parameters. On the down side, computing an actual estimate for d is not straightforward. For example, the MLE produces trivial values that are of scarce utility: $\hat{d}_{\text{ML}} = 0$ when $C_{01} = 1$, and $\hat{d}_{\text{ML}} = +\infty$ when $C_{01} = 0$.

In practice, some other solution will be used to translate the values (4) into actual distance estimates. Interestingly, as shown in the following sections, assuming a particular scheme is not necessary. Localization approaches using RSS and connectivity data can be compared on a general basis using the Fisher information and the CRB.

2.4 Fisher Information

The Fisher information (F) measures the amount of information that a random variable carries about an unknown parameter. Here, the random variables are the ones defined by (1) and (4), while the parameter to estimate is d. The inverse of the Fisher information, known as the Cramér–Rao Bound, is the minimum variance that can be achieved when estimating d using *any* unbiased estimator:

$$\text{Var}\{\hat{d}\} \geq \frac{1}{F(d)}. \tag{5}$$

By comparing the Fisher information for RSS and connectivity measurements, we can identify under which conditions the minimum theoretical error for one approach is lower than the other. In turn, this allows us to select which localization technique can potentially produce the lowest error.

2.4.1 RSS Measurements

Details to compute the Fisher information are presented in [8] for RSS estimates and in [9] for proximity information in the 2D case. For the two nodes in Figure 1, the Fisher information associated with RSS measurements is:

$$F_{\text{rss}}(d) = K_{\text{c}}^2 \, \frac{1}{d^2}, \tag{6}$$

where the constant $K_{\text{c}} = (10 n_{\text{p}})/(\sigma_{\text{dB}} \log 10)$ depends on the parameters of the propagation model.

Figure 2a shows F_{rss} as a function of d for different values of n_{p} and σ_{dB}. The plots describe what was already seen in (3): the amount of information available to estimate d decreases for increasing values of the distance and increasing values of the ratio $\sigma_{\text{dB}}/n_{\text{p}}$. In particular, when $\sigma_{\text{dB}}/n_{\text{p}}$ increases, the estimates become less accurate because the variability caused by RF shadowing "blurs" the RSS measurements, decreasing their dependence on the distance.

2.4.2 Connectivity

In the case of connectivity data, the Fisher information depends not only on the distance between the two nodes, but also on the value of the threshold P_{th} used in (4). To present an expression of F similar to (6), the value P_{th} is

[1] Averaging the measured values reduces part of the variability caused by multi-path propagation of the RF signal impinging on static and moving obstacles.

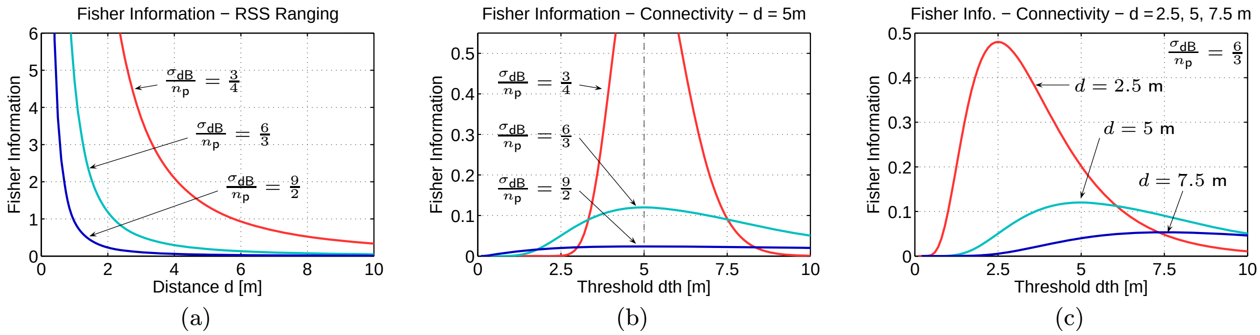


Figure 2: Fisher Information for RSS and connectivity measurements.

converted into a threshold distance:

$$d_{\text{th}} = d_0 10^{(P_0 - P_{th})/(10\, n_p)}. \tag{7}$$

Using d_{th}, which is the MLE for P_{th}, the Fisher information can be written as:

$$F_{\text{conn}}(d, d_{\text{th}}) = K_{\text{c}}^2 \, \frac{1}{d^2} \, I_{\text{r}}(d, d_{\text{th}}). \tag{8}$$

The equation above show that F_{conn} is equal to F_{rss} multiplied by an additional term I_{r}. The term I_{r} depends on the ratio between d and the threshold distance d_{th}:

$$I_{\text{r}}(d, d_{\text{th}}) = \frac{2}{\pi} \frac{\exp\left[-K_{\text{c}}^2 \log(d/d_{\text{th}})^2\right]}{1 - \text{erf}\left[K_{\text{c}} \log(d/d_{th})/\sqrt{2}\right]^2} \quad . \tag{9}$$

Figure 2b shows F_{conn} as a function of d_{th} when $d = 5\,\text{m}$ for different values of $\sigma_{\text{dB}}/n_{\text{p}}$, and Figure 2c shows F_{conn} for increasing distances d. The F_{conn} value always peaks when $d_{\text{th}} = d$. Connectivity measurements reach the maximum information content when P_{th} corresponds to a threshold distance d_{th} equal to the true node distance (which is unknown). If $d_{\text{th}} = d$, the probability of detecting the node as connected is 0.5, and F_{conn} is approximately 37% lower than F_{rss}. In fact, $I_{\text{r}}(d, d_{\text{th}}) = 2/\pi \cong 0.63$ for $d_{\text{th}} = d$.

2.4.3 Discussion

Comparison between F_{rss} and F_{conn} shows that RSS measurements always carry greater information content than connectivity ones. However, this is only true as long as the nodes are within the *radio range* of each other.

When nodes are within each other's radio range, they can successfully exchange radio messages and p_{01} can be computed by averaging the values $\{z_1, z_2, z_3, \ldots\}$. The value p_{01} can be used for range estimates using (2), or it can be used to derive connectivity information using (4). Depending on the choice of P_{th}, two nodes that are within each other's radio range can be considered connected or disconnected.

On the other hand, when nodes are out of range, they will not be able to communicate and no RSS information will be collected. In this case, a range-based approach such as the MLE will not be able to produce any position estimate (i.e. F_{rss}= 0). Instead, if a connectivity scheme is used, the occurrence of nodes that are out of range can be associated to the value $C_{01} = 0$, therefore a position estimate is still possible (F_{conn}> 0).

The diverse nature of the measurements implies a fundamental difference between the two approaches. RSS ranging is more accurate when nodes are in the radio range of each other, but a connectivity scheme is naturally suited to localize nodes that are unable to communicate.

3. NETWORK LOCALIZATION

In typical applications, localization involves computing the positions of multiple nodes deployed as a network. For a system with N nodes in a 2D space, the information to estimate the node positions is measured by a $2N \times 2N$ matrix known as the *Fisher Information Matrix* (FIM)[2] [8, 9]. The CRB, which is obtained by inverting the FIM, sets a lower bound on the covariance matrix of the unknown parameters. In our analysis, we use CRB_{rss} and CRB_{conn} to denote the average value of the $2N$ coordinates' standard deviation.

3.1 CRB Analysis

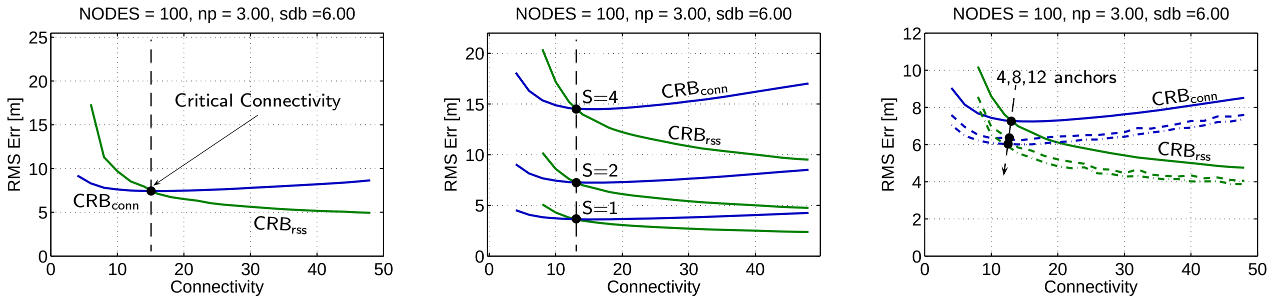
Figure 3a shows the CRB_{rss} and CRB_{conn} computed for a 100 node network with four anchor nodes on the corners of the deployment area. We consider different connectivity values obtained by increasing the communication range of each node.

As shown by the plots, the CRB_{rss} monotonically decreases with the connectivity. In the case of RSS ranging, a given connectivity value, say ten, means that each node is in the radio range of other ten nodes; hence, ten range estimates are available to compute its position. As the connectivity increases, the number of measurements increases, causing the CRB_{rss} to decrease.

Differently from the CRB_{rss}, the CRB_{conn} does not decrease with the connectivity. In fact, in the case of connectivity data, a connectivity value equal to ten means that the average RSS of ten nodes are above the threshold P_{th}. The remaining 89 nodes are considered disconnected, either because their signal strength is below the threshold or because they are out of range. In any case, the number of measurements available is equal to 99. Therefore, increasing the connectivity does not necessarily cause the CRB_{conn} to decrease.

To understand the effect of different d_{th} values on the CRB_{conn}, we recall that F_{conn} reaches its maximum when the distance between the nodes is approximately equal to

[2] The FIM's elements are similar to (6) and (8), but the coordinates (x_i, y_i) of the nodes are explicitly considered in the equations of F_{rss} and F_{conn}[8, 9]. Equations (6) and (8), however, suffice to understand the analysis presented in the next sections.



(a) (b) (c)

Figure 3: CRBs for a 100 node network

$d_{\rm th}$. When a particular threshold is used, only the nodes whose separation distance is similar to $d_{\rm th}$ will contribute a significant amount of information. As shown in Figure 4, increasing the threshold, and therefore increasing the connectivity, increases the number of nodes whose distance is similar to $d_{\rm th}$. We recall, however, that $F_{\rm conn} \propto 1/d^2$, therefore these nodes contribute individually less information as the distance increases. In conclusion, the choice of $d_{\rm th}$ determines a tradeoff between obtaining high-quality measurements from a few nearby nodes, or obtaining less valuable data for a larger number of nodes that are farther away.

Analysis of the $\rm CRB_{conn}$ also explains why connectivity-based schemes achieve poor performance when the network connectivity is too high, or worse, the network is fully connected. This situation arises when the nodes' communication range is comparable to the dimensions of the deployment area. In this case, if $d_{\rm th}$ is increased excessively, each node will be connected to most of the other nodes, but only a few boundary nodes will be at a distance similar to $d_{\rm th}$. Since these nodes contribute only a limited amount of information, the estimation error will be generally high. The effect of large $d_{\rm th}$ values is visible in Figures 5b,c for the case of the 49 node networks. The plots show that the $\rm CRB_{conn}$ rapidly increases when the connectivity reaches values close to the total number of nodes.

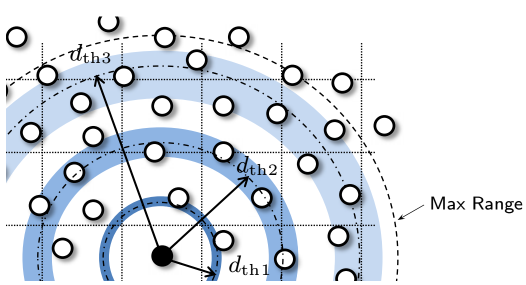


Figure 4: Effect of choosing a different threshold. Nodes with distances close to the threshold contribute the most information.

3.2 Critical Connectivity

In Figure 3a we observe the existence of a *critical connectivity* ($c_{\rm cr}$) value where the two CRB lines cross. *For connectivity values below c_{cr}, CRB_{conn} is lower than CRB_{rss}, implying that connectivity-based localization is potentially more accurate than RSS ranging, while the opposite is implied for values above c_{cr}.*

Given a network to localize, we will choose which approach to use based on comparison between the actual network connectivity and the critical connectivity value. Unfortunately, computing the CRBs requires knowledge of the true node positions, therefore we cannot use this approach in practical applications. To determine an alternative solution, first we will study some properties of the $c_{\rm cr}$ value, and then we will show how to approximate it without knowing the actual node positions.

3.3 Properties of the Critical Connectivity

We find that the $c_{\rm cr}$ value does not change with the node density and the number of anchor nodes.

The independence of $c_{\rm cr}$ from the node density derives from the structure of the terms $F_{\rm rss}$ and $F_{\rm conn}$, which scale with the distances between the nodes. Assume all the node distances are scaled by a factor S. Also assume that $d_{\rm th}$ is scaled by the same factor, so the network connectivity remains constant. The relative position of $\rm CRB_{rss}$ and $\rm CRB_{conn}$ will not change because the FIMs will be multiplied by the same constant factor. In fact:

$$\begin{aligned} F_{\rm rss}(Sd) &= S^{-2}F_{\rm rss}(d) \\ F_{\rm conn}(Sd, Sd_{\rm th}) &= S^{-2}F_{\rm conn}(d, d_{\rm th}). \end{aligned} \qquad (10)$$

Figure 3b shows the position of $c_{\rm cr}$ when the network coordinates are scaled by a factor $S = \{1, 2, 4\}$.

Our simulations also show that increasing the number of anchor nodes causes both of the CRBs to decrease, but without significantly affecting the position of $c_{\rm cr}$. This property is visible in Figure 3c, which shows the CRBs for a 100 node topology with 4,8, and 12 anchors.

On the other hand, the $c_{\rm cr}$ value increases with the network size. According to the analysis in previous sections, the $\rm CRB_{rss}$ depends only on the nodes that are in radio range, while the $\rm CRB_{conn}$ depends on the total number of nodes in the network. Augmenting the network size (while maintaining the same network connectivity) increases the number of connectivity measurements available to estimate the position of each node. Therefore, when the number of nodes increases, the $\rm CRB_{conn}$ decreases with respect to the $\rm CRB_{rss}$, causing the the $c_{\rm cr}$ value to increase. This property can be observed in Figures 5b and c. In both cases, the $c_{\rm cr}$ value for a 100 node network is greater than the $c_{\rm cr}$ value computed for a similar network with only 49 nodes.

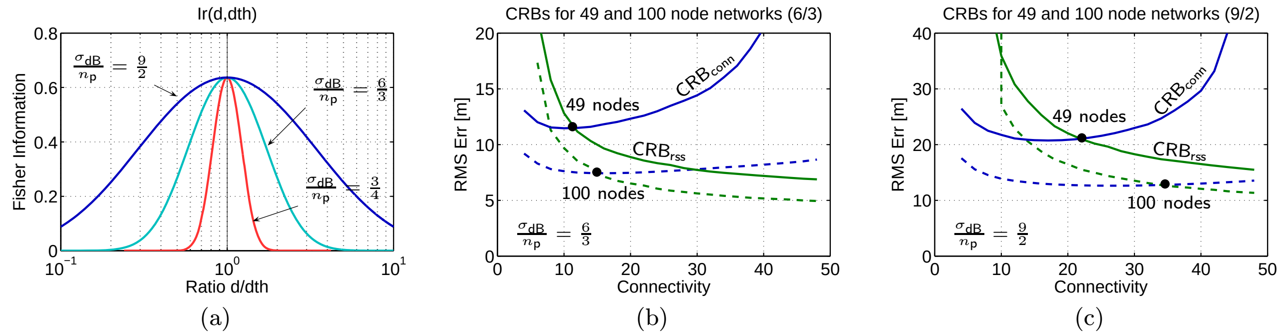


Figure 5: Effect of increasing the number of nodes and the ratio $\sigma_{\text{dB}}/n_{\text{p}}$ on the critical connectivity.

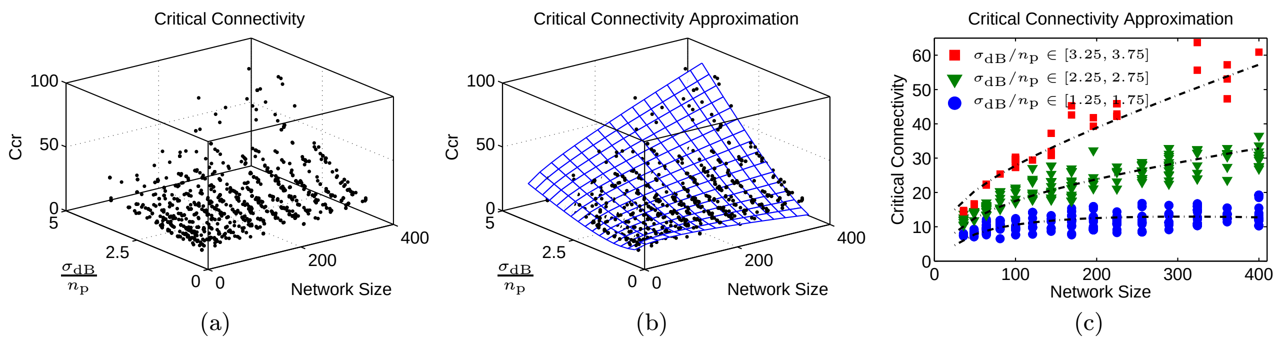


Figure 6: Simulation results for critical connectivity values and their approximation.

The c_{cr} value also increases when the ratio $\sigma_{\text{dB}}/n_{\text{p}}$ increases. The term K_{c}^2 that multiplies both F_{rss} and F_{conn} will decrease by the same amount, however, in the case of F_{conn} some of the loss is compensated by term I_{r}. As shown in Figure 5a, this term increases with the ratio $\sigma_{\text{dB}}/n_{\text{p}}$, therefore the CRB_{conn} will increase less than the CRB_{rss}. Similarly to the previous case, the c_{cr} value will increase. Comparison between Figures 5b and 5c illustrates the property described above. When the ratio $\sigma_{\text{dB}}/n_{\text{p}}$ is increased from 6/3 dBm to 9/2 dBm, both of the c_{cr} values computed for a 49 and a 100 node network increase.

3.4 Critical Connectivity Approximation

We have seen that the c_{cr} value increases with the network size and with the ratio $\sigma_{\text{dB}}/n_{\text{p}}$. In this section, we use extensive simulations to model the dependence of c_{cr} on these two parameters.

We generated about 1300 topologies with the number of nodes between 25 and 400. The nodes are placed using a *noisy grid* deployment model with various levels of grid perturbation. Four nodes on the corners of the network are used as anchors. For each topology, the parameters n_{p} and σ_{dB} are sampled from the intervals [2, 4] and [3, 9] dBm respectively, resulting in values of the ratio $\sigma_{\text{dB}}/n_{\text{p}}$ between 0.75 dBm and 4.5 dBm.

Figure 6a shows the simulation results. The c_{cr} values are plotted against the simulation parameters and appear to lie on a smooth surface. We interpolate the c_{cr} values using a function that is empirically found:

$$\begin{aligned}\tilde{c}_{\text{cr}}(n,r) &= a_0 + a_1 n + a_2 r + a_3 nr + \\ &\quad + a_4 \log n + a_5 \exp(-r), \qquad (11)\end{aligned}$$

where n is the number of nodes and r is the value of $\sigma_{\text{dB}}/n_{\text{p}}$. The values of the coefficient a_i, obtained by least squares fitting, are: $a_0 = -30.8305, a_1 = -0.0892, a_2 = 7.5626, a_3 = 0.0475, a_4 = 5.2669$, and $a_5 = 33.5086$.

The squared error between $\tilde{c}_{\text{cr}}(n,r)$ and the data points is equal to 5.72, while the average error is equal to 1.79. We find this error sufficiently small for practical application of (11) in approximating the c_{cr} value. Figure 6b shows the interpolating surface (11) together with the data point. Figure 6c shows the c_{cr} values for different intervals of the ratio $\sigma_{\text{dB}}/n_{\text{p}}$. The dotted lines are computed using (11) for r equal to the central value of the $\sigma_{\text{dB}}/n_{\text{p}}$ ranges considered.

As seen in Figure 6c, for low values of $\sigma_{\text{dB}}/n_{\text{p}}$, the c_{cr} value stabilizes around ten. As the ratio $\sigma_{\text{dB}}/n_{\text{p}}$ increases, however, there is an higher correlation between network size and c_{cr} values, therefore range-based schemes are beneficial only in highly connected networks. These results confirm the observations of other authors, who have occasionally noted that connectivity-based schemes outperform range-based ones in conditions of low connectivity [2] or when the ranges are estimated using noisy measurements [1, 7].

4. TEST CASE

Consider the 100 node network of Figure 7 with parameters $n_{\text{p}} = 3$ and $\sigma_{\text{dB}} = 8$ dBm. Applying (11), we find: $\tilde{c}_{\text{cr}}(100, 8/3) = 19.7$ (the exact value found using the two

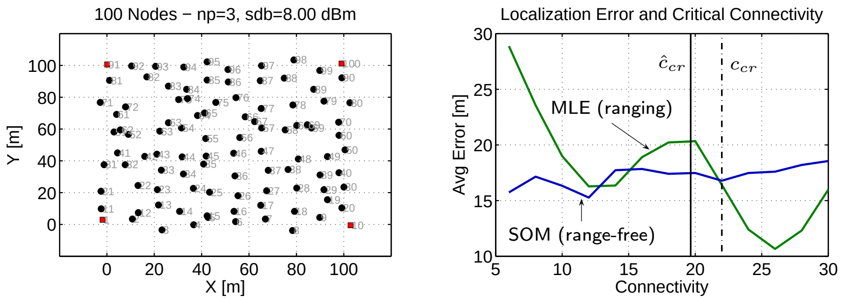


Figure 7: 100 node network test case.

CRBs is 22). According to our analysis, we will use a connectivity based scheme for connectivity values below 19.7, and a range-based scheme when the network's connectivity is above 19.7.

To validate our choice, we compute the node positions using two algorithms. The first one is a range-free localization scheme based on Self-Organizing Maps that we described in [3]. We choose this scheme because it has shown to perform well for low connectivity values. The other one is a range-based scheme that computes the MLE using gradient descent[3] [8]. As shown in Figure 7, *a posteriori* analysis of the error confirms the choice made by using (11). For connectivity values lower than 20, the range-free scheme's error is lower than the MLE's error; the opposite is true for connectivity above 20.

5. CONCLUSIONS

We have presented an analysis of the conditions under which a scheme that uses range estimates obtained from RSS values can potentially perform better than one that uses connectivity measurements and vice versa. We have also shown how the choice of the scheme to use is based on comparing the connectivity of the network to localize against the critical connectivity value discussed in Section 3.2. This value can be approximated using a function of the network size and the ratio $\sigma_{\mathrm{dB}}/n_{\mathrm{p}}$.

Our conclusions are based on analysis of the theoretical bounds for the localization error. For connectivity values below c_{cr}, not every range-free scheme will perform better than every range-based scheme. Similarly, schemes that use RSS range estimates may perform worse than range-free schemes for connectivity above c_{cr}. However, if the schemes considered are known to perform close to the CRBs, analysis of the critical connectivity will provide valuable information to help choosing between them.

We note that our results are valid for schemes that use only range estimates or connectivity information. Additional information can be used with both of the approaches. For example, range-based schemes can impose constraints on the minimum separation distance between disconnected nodes (e.g. [4]). Similarly, connectivity-based schemes can use RSS information to "sort" one-hop neighbors [5]. In both cases, using additional information will cause the localization error to decrease, and the values computed using (11) will not necessarily correspond to the intersection point of the two CRBs. Analysis of these cases will be the focus of our future research work.

[3]We use the output of the range-free scheme as initial position for the gradient descent.

6. ACKNOWLEDGEMENTS

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