

# On Maximizing Network Lifetime of Broadcast in WANETs under an Overhearing Cost Model

Guofeng Deng and Sandeep K.S. Gupta

Arizona State University, Tempe, AZ 85281, USA  
{guofeng.deng, sandeep.gupta}@asu.edu

**Abstract.** Absence of line power supplies imposes severe constraints on nodes in wireless ad hoc and sensor networks. In this paper, we concentrate on finding a broadcast tree that maximizes the network's lifetime. Previous studies showed that this problem is polynomially solvable when assuming the receiver consumes no energy or only the designated receiver consumes energy for receiving packets. Due to the broadcast nature of the wireless medium, however, unintended active nodes in the receiving range of a transmitting node may overhear the message and hence contribute to energy wastage. Under the overhearing cost model, the problem becomes NP-hard and the approximation ratio of the existing solutions under the non-overhearing cost model can be as bad as  $\Omega(n)$ . We investigate the problem by developing heuristic solutions. Simulation results show that our algorithms outperform the existing algorithms by up to 100%.

## 1 Introduction

Broadcast is an essential networking primitive in Wireless Ad hoc NETWORKS (WANETs) with a wide range of applications such as software updating, teleconferencing and on-line gaming. In particular, it is widely applied to command and query distribution in Wireless Sensor Networks (WSNs). However, the limited energy battery-powered wireless nodes impose severe energy constraints. This mandates efficient usage of energy resources for all the computation and communication tasks including a network-wide broadcast, as fast energy drain-out may lead to network partition and short network lifetime. Maximizing the lifetime of a broadcast operation is, therefore, imperative for improved availability of broadcast services.

One of the efficient and most widely used broadcast structure in WANETs is the tree-based structure. We adopt the definition of network lifetime of broadcast as the duration in which the source node is capable of transmitting broadcast packets to each node in the network. Approaches to maximizing tree based broadcast network lifetime fall into two categories: *static* [1] [2] [3] [4] [5] and *dynamic* [6] [7]. In a static approach, a single broadcast tree is used throughout the broadcast session, i.e. the broadcast tree is fixed once formed. In a dynamic approach, a series of broadcast trees is employed one after another, i.e. broadcast trees are updated during the broadcast session. Essentially, the latter may improve the fairness of battery energy consumption in each node and hence prolong the network lifetime. However, control overhead for frequent information exchange and network wide synchronization may impact potential lifetime increment.

We concentrate on the static approach to the problem of maximizing network lifetime of broadcast, or equivalently *Maximizing Broadcast Tree Lifetime* (MaxBTL). The problem was addressed before by omitting receiving cost. In reality, however, a receiver’s power consumption is not negligible [8] [9] [10] [11]. It was further studied in a model, which assumed that only designated receivers consume energy for receiving packets. In reality, however, this model relies on the synchronization among neighboring nodes, i.e. at any time only the transmitter and its designated receivers are active, but all other nearby nodes are switched off to avoid overhearing. We refer to the above model, under which the receivers consume no energy or only designated receivers consume energy for receiving packets as *Non-Overhearing Cost* (NOC) model.

In this paper, we study the MaxBTL problem under a power consumption model in which a node consumes a certain amount of energy for receiving a packet either as a designated or non-designated receiver. This model is referred to as the *Overhearing Cost* (OC) model. While it is polynomially solvable under the NOC model, the MaxBTL problem becomes NP-hard due to the consideration of overhearing cost. We show that the two optimal solutions under the NOC model can perform  $\Omega(n)$  times worse than the optimal solution under the OC model. We then propose two greedy heuristic solutions, which take into account the overhearing cost when generating a broadcast tree. Simulation results show that they outperform the existing solutions. In particular, the performance of PRP is better than that of TPO and DRP by up to 100%.

The rest of the paper is organized as follows. Section 2 briefly summarizes related work. Section 3 introduces the network model and formulates the optimization problem. Section 4 investigates heuristic solutions to the MaxBTL problem under the OC model. Section 5 reports simulation results. Finally, Section 6 concludes the paper.

## 2 Related Work

Camerini proved that a Minimum-weight Spanning Tree (MST) minimizes the maximum link weight among all the spanning trees in an undirected graph [12]. Its application to a WANET, which is modeled as an undirected graph, includes that a MST minimizes the maximum transmission power among all the broadcast trees. In a special case, where homogenous nodes carry identical batteries, the lifetime of a node is inversely proportional to its transmission power and hence a MST is a maximum lifetime broadcast tree [3]. Das et al. extended the result to a network that is composed of nodes with various battery capacities and proposed a minimum decremental lifetime (MDLT) algorithm [4]. Lloyd et al. and Floréen et al. sought a subnetwork of maximum lifetime in which the source node is connected to all the broadcast group members [1] [2]. It is easy to see that any broadcast tree contained in such a subnetwork has the maximum lifetime. All above solution assumed that the receiver consumes no energy for receiving packets. In [5], we proposed a polynomial optimal solution to the problem of MaxBTL under the assumption that only the designated receivers consume receiving power. In this paper, we take into account the receiving cost of non-designated receivers as well.

Recently, some topology control methods have been developed with the goal of minimizing interference [13] [14]. The interference of a link is evaluated in terms of the number of nodes that are affected by the bidirectional transmission between the two

incident nodes in the resulting subnetwork; the interference of a node is defined as the maximum link interference of all the links incident to it. Interference-aware topology control techniques intend to minimizing the maximum or the average link or node interference in the resulting subnetwork. In this paper, however, we are interested in minimizing the number of nodes which overhears the transmission along with the transmission cost. The two problems are different because high interference does not necessarily mean high overhearing cost and vice versa. For example, a subnetwork, in which a link incident to the source node covers all the nodes in the network, has high interference, but it involves no overhearing cost during the tree based broadcast, for which the source node directly transmits packets to all the nodes in the network.

### 3 Preliminaries

#### 3.1 Network Model

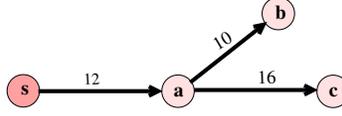
In WANETs, a node may act as a transmitter, a receiver or both. The *power consumption* in milliWatts (mW) of the transceiver of a node  $u$ , denoted by  $p_u$ , is the sum of transmission power (due to the amplifier in the transceiver), receiving power, and energy expenditure in the transmitter circuit electronics (e.g. multiplexing), denoted by  $p_u^{\text{tr}}$ ,  $p_u^{\text{rc}}$ ,  $p_u^{\text{ce}}$  respectively, i.e.  $p_u = p_u^{\text{tr}} + p_u^{\text{rc}} + p_u^{\text{ce}}$ . In particular,  $p_u^{\text{rc}} = 0$  if  $u$  does not receive any packet from other nodes, e.g. the source node in a multicast tree;  $p_u^{\text{tr}} = p_u^{\text{ce}} = 0$  if  $u$  does not forward any packet to other nodes, e.g. a leaf node in a multicast tree.

A signal can be successfully detected if the signal strength at the receiver is above a certain level after traversing the fading channel. For any pair of nodes  $u$  and  $v$ , we define  $p(u, v)$  as the *transmission power threshold* (TPT) of  $u$  being successfully received by  $v$ . In other words, the transmission from  $u$  to  $v$  necessitates  $p_u^{\text{tr}} \geq p(u, v)$ . Notice that we do not require that  $p(u, v) = p(v, u)$ . In the wireless medium, a single transmission can be received by multiple receivers. This effect that assists in conserving energy is referred to as *Wireless Multicast Advantage* [15]. To reach a set  $C$  of neighbors,  $p_u^{\text{tr}}$  of node  $u$ , which uses an antenna of appropriate directionality, is set to the maximum TPT to any node in  $C$ , i.e.  $p_u^{\text{tr}} = \max_{v \in C} p(u, v)$ .

Motivated by [10], we present the OC model as follows. A node consumes a fixed amount of energy for receiving a packet from any other node. Hence, for each broadcast packet, the receiving cost of a node  $v$  is determined by the number of copies that  $v$  receives from its parent node as well as other neighboring nodes. Alternatively, we consider the receiving power  $p_v^{\text{rc}}$  of node  $v$  is proportional to the number of receivable neighbors around  $v$ , i.e.  $p_v^{\text{rc}} = \hat{p}_v^{\text{rc}} \sum_{u \in N} X(u, v)$ , where  $\hat{p}_v^{\text{rc}}$  is a unit of receiving power of  $v$ ,  $N$  is the set of nodes in the network and  $X(u, v) = 1$  if  $v$  receives a packet from  $u$ , i.e.,  $p_u^{\text{tr}} \geq p(u, v)$ , or  $X(u, v) = 0$  otherwise. In this paper, we assume  $\hat{p}_v^{\text{rc}}$  to be a node-dependent constant value.

For example, Fig. 1 depicts a broadcast tree rooted at the source node  $s$  in a WANET, where the TPT between each pair of nodes is symmetric. Under the OC model, node  $s$  can overhear the transmission by node  $a$  to  $b$  and  $c$ . Hence,  $p_s = p(s, a) + \hat{p}_s^{\text{rc}} + p_s^{\text{ce}}$ ; while under the NOC model,  $p_s = p(s, a) + p_s^{\text{ce}}$ .

We model a WANET as a directed and link-weighted graph  $G = (V, A, p)$  called a *Transmission Power Graph* (TPG), where  $V$  is a set of nodes,  $A$  is a set of links



**Fig. 1.** A broadcast tree rooted at node  $s$ . An arrow that starts from a transmitter and ends in a receiver is associated with the transmission power threshold in milliWatts.

and  $p : A \rightarrow \mathbb{R}^+$  is a weight function. For each pair of nodes  $u, v \in V$ , there is a link in  $A$  joining  $u$  and  $v$ , denoted by  $(u, v)$ , if  $\mathcal{P}_u^{\text{tr}} \geq p(u, v)$ , where  $\mathcal{P}_u^{\text{tr}}$  is a node-dependent constraint called the *maximum transmission power*. The weight of a link  $(u, v)$  assigned by the function  $p$  is the TPT between  $u$  and  $v$ , i.e.  $p(u, v)$ . A node  $u$  is capable of adjusting its transmission power up to  $\mathcal{P}_u^{\text{tr}}$ . While “broadcast tree” is a term used in networks, “spanning tree” is the counterpart in graph theory.

### 3.2 Battery Life

We denote the *battery capacity* in Watt-Hour (WH) of a node  $u$  by  $E_u$ , which may vary from node to node. A battery is treated as a linear storage of current and remains functional until the rated capacity of energy has been dissipated [8]. For example, the *lifetime* in hours (H) of a node  $u$ , denoted by  $\ell_u$ , is the ratio between  $E_u$  and its power consumption  $p_u$ , i.e.  $\ell_u = \frac{E_u}{p_u}$ , since the transceiver is the dominant power consumer during the system operation in a wireless node [16]. We note that the lifetime computed under the linear model does not necessarily reflect the actual period of time in which the battery is functional [17]. However, the essence of maximizing network lifetime of broadcast is to balance the energy consumption at each node. In this sense, the linear model allows us to concentrate on how to improve the fairness. We put into our future consideration the effects of non-linear factors on the battery lifetime [18, 19]. In the rest of the paper, we assume that a node is reliable in the sense that it dies only in the case of depletion of the battery energy.

### 3.3 Problem Statement

The *lifetime of a broadcast tree*  $T$ , denoted by  $\mathcal{L}(T)$ , is the period of time until the first node in  $T$  fails, i.e.  $\mathcal{L}(T) = \min_{u \in T^n} \ell_u$ , where  $T^n$  is the set of nodes in  $T$  [1–4]. In this paper, given a WANET  $W$ , which consists of stationary battery-powered nodes, and a source node  $s$ , we investigate the problem of MaxBTL, which is to seek a broadcast tree  $T^*$  such that  $\mathcal{L}(T^*) = \max_{T \in \mathcal{T}} \mathcal{L}(T)$ , where  $\mathcal{T}$  is the set of broadcast trees rooted at  $s$  in  $W$ . Notice that the network lifetime of simultaneous multi-session broadcast is more complicated and beyond the discussion in this paper.

Overhearing cost makes the problem difficult because considering transmission power level exclusively does not suffice. In some cases, a node not transmitting but only receiving packets from nearby nodes dies earlier than nodes that transmit packets. Actually, the MaxBTL problem under the OC model is NP-hard because a special case

of the problem is equivalent to the minimum set cover problem, which is known to be NP-hard [20]. We omit the proof due to space limitation.

It is easy to see that if each node has an identical level of battery energy, the MaxBTL problem is equivalent to finding a tree that minimizes the maximum nodal power consumption among all the broadcast trees. In this paper, we assume identical battery capacity for presentation simplicity. However, the solutions presented in the rest of the paper are applicable to the non-identical battery scenarios after minor modification, e.g. replacing nodal power consumption by node lifetime as link metric.

## 4 Heuristic Solutions

In this section, we first summarize the two optimal algorithms, namely **Transmission Power Only** (TPO) and **Designated Receiver Power** (DRP), to the MaxBTL problem under the NOC model. Then, we propose two greedy heuristic solutions, **Cumulative Designated Receiver Power** (CDRP) and **Proximity Receiver Power** (PRP).

### 4.1 TPO and DRP Algorithms

Given a TPG, TPO and DRP iteratively grow a spanning tree that is rooted at the given source node. They start from a single node tree that consists of only the source node. In each iteration that follows, a link, which has the minimum “weight” among all the links that join an in-tree node and a not-in-tree node, is included in the tree.

We present TPO in Algorithm 1. In Line 1, tree  $T$  is initialized to a single-node tree that consists of node  $s$  only. In each iteration of the *while* loop in Lines 2 through 10, a link is included into  $T$  until  $T$  spans every node in the network. The *for* loop in Lines 4 through 8 is used to choose a link, which joins an in-tree node and a not-in-tree node, that has the lowest weight, i.e. the TPT. In a TPG consisting of  $n$  vertices and  $m$  links, the *while* loop is executed  $n$  times and the *for* loop runs  $O(m)$  times in each iteration of the *while* loop. Therefore, the time complexity of TPO is  $O(mn)$ .

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#### Algorithm 1 Transmission Power Only (TPO)

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**Input:** A TPG  $G = (V, A, p)$  and a node  $s \in V$

**Output:** A spanning tree  $T = (V', A')$

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1:  $V' \leftarrow \{s\}; A' \leftarrow \phi$ 
2: while  $|V'| < |V|$  do
3:    $p(u^0, v^0) \leftarrow \infty$   $\{(u^0, v^0)$  is the link to be included in  $T\}$ 
4:   for all  $u \in V'$  and  $v \in V \setminus V'$  and  $(u, v) \in A$  do
5:     if  $p(u^0, v^0) > p_u^{ce} + p(u, v)$  then
6:        $u^0 \leftarrow u; v^0 \leftarrow v$ 
7:     end if
8:   end for
9:    $V' \leftarrow V' \cup \{v\}; A' \leftarrow A' \cup \{a\}$ 
10: end while
11: return  $T$ 

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Different than TPO, which omits all receiving cost, DRP takes into account the effects of receiving cost in the designated receivers on the network lifetime as well. The metric of a link  $(u, v)$ , denoted by  $w(u, v)$ , used in DRP is defined as the higher power consumption of transmitter  $u$  and receiver  $v$ , i.e.

$$w(u, v) = \begin{cases} \max \{p(u, v) + p_u^{\text{ce}}, \hat{p}_u^{\text{rc}}\}, & \text{if } u \text{ is the source node,} \\ \max \{p(u, v) + p_u^{\text{ce}} + \hat{p}_u^{\text{rc}}, \hat{p}_v^{\text{rc}}\}, & \text{otherwise.} \end{cases} \quad (1)$$

DRP has similar steps as TPO such that TPO becomes DRP if we replace any appearance of link weight function  $p$  in Algorithm 1 by  $w$ . The time complexity of DRP is  $O(mn)$ , where  $n$  and  $m$  are the number of vertices and arcs, respectively, in a TPG.

TPO and DRP are optimal solutions to the MaxBTL problem under the assumption of no receiver cost and designated receiver cost, respectively [5]. However, we show that they can perform as bad as  $\Omega(n)$  times of the optimal value under the OC model in Theorem 1, where  $n$  is the network size.

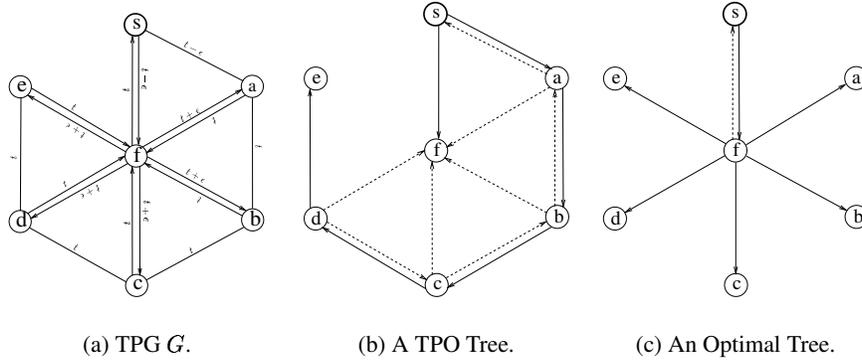
**Theorem 1.** *As heuristic solutions to the MaxBTL problem under the OC model, TPO and DRP have an approximation ratio that can be as bad as  $\Omega(n)$ , where  $n$  is the network size.*

*Proof.* Consider a TPG  $G = (V, A, p)$  depicted in Fig. 2(a), where  $s$  is the source node and the rest are destinations. Each link is associated with its TPT. Let  $\hat{p}_u^{\text{rc}} = t$  and  $p_u^{\text{ce}} = 0$  for each node  $u \in V$ . Let  $\epsilon$  be a sufficiently small positive real number. We first examine the result of TPO. Initially,  $s$  is the only node in  $T$ ; then links  $(s, a)$  and  $(s, f)$  are picked because they both have the lowest TPT, i.e.  $t - \epsilon$ ; in the third iteration, links that join an in-tree node and a not-in-tree node include  $(a, b)$ ,  $(f, b)$ ,  $(f, c)$ ,  $(f, d)$  and  $(f, e)$ . Among them,  $(a, b)$  has the lowest weight  $t$  compared to the weight of all other links, which is  $t + \epsilon$ , and hence is included in  $T$ . Similarly, links  $(b, c)$ ,  $(c, d)$  and  $(d, e)$  are included in that order. In the resulting tree shown in Fig. 2(b), each node on the border, except for node  $e$ , is a transmitting node and is overheard by node  $f$  located in the center. Therefore, the maximum power consumption is  $5t$  by node  $f$  in the resulting tree. Fig. 2(c) shows an optimal solution for the given TPG, in which the node in the center acts a transmitting node and forwards packets to all the node on the border. The maximum power consumption in this optimal solution is  $t + t = 2t$  irrespective of the size of the network. In a generic network, we can enlarge the network depicted in Fig. 2(a) by allocating more nodes on the border. If  $n$  is the network size, then the maximum power consumption of a spanning tree constructed by using TPO is  $(n - 2)t$ . So, the approximation ratio of TPO is at least  $(n - 2)/2$ , i.e.  $\Omega(n)$ . A similar analysis will show that DRP has an approximation ratio of  $\Omega(n)$  under the OC model.

## 4.2 CDRP Algorithm

A common problem with both of the above algorithms is that the effects of receiving power because of overhearing is not considered. We present CPRP, an extension to DRP, that takes into account overhearing cost. When choosing a link in each iteration of CDRP, the sender's, say node  $u$ , receiving power  $p_u^{\text{rc}}$  is taken into account. In other words, the weight of a link  $(u, v)$  is defined as

$$w(u, v) = \max \{p_u^{\text{ce}} + p(u, v) + p_u^{\text{rc}}, \hat{p}_v^{\text{rc}}\} . \quad (2)$$



**Fig. 2.** Theorem 1. A solid line represents two directional link toward opposite directions. A dotted arc represents an overhearing transmission. Each link is associated with its TPT.

We present CDRP in Algorithm 2. For each node  $u$ ,  $p_u^{rc}$  and  $p_u^{tr}$  are initialized to 0. In each iteration of the *while* loop, a link that has the lowest weight and joins an in-tree node and a not-in-tree node is included in  $T$ . The effects of the newly added link, say  $(u, v)$ , on nearby receivers are computed, i.e. a node  $k$  will have some additional cost  $\hat{p}_k^{rc}$  for receiving packets transmitted by  $u$  if  $k$  can hear  $u$  because of transmission  $(u, v)$  but  $k$  is not covered by previous transmission from  $u$  (see Lines 14 through 16). This condition can avoid overcounting  $k$ 's receiving power. In a TPG consisting of  $n$  nodes and  $m$  links, the *while* loop runs  $n$  times and each *for* loop runs  $O(m)$  times. Therefore, CDRP has the time complexity of  $O(mn)$ , which is same as TPO and DRP.

### 4.3 PRP Algorithm

For each link, CDRP considers only the sender's receiving power. We propose PRP, which takes into account the effect of newly added links on nearby receivers. This is because overhearing receivers may have the maximum power consumption. The weight of a link  $(u, v)$  is defined as

$$w(u, v) = \max \{ p_u^{ce} + p(u, v) + p_u^{rc}, p_k^{ce} + p_k^{tr} + p_k^{rc} + \hat{p}_k^{rc}, p_l^{te} + p_l^{tr} + p_l^{rc} \}, \quad (3)$$

where  $k \in V$  satisfies  $p(u, k) \leq p(u, v)$  and  $p(u, k) > p_u^{tr}$ , and  $l \in V$  satisfies  $p(u, l) \leq p(u, v)$  and  $p(v, l) \leq p_l^{tr}$  ( $p_l^{ce} = 0$  for any non-transmitting node  $l$ ). In other words, the weight of a link  $(u, v)$  is the maximum power consumption after including  $(u, v)$  in the tree, which is either by the transmitter  $u$  or the affected nearby receivers of  $u$ . Notice that an additional cost is added to a node  $k$  only if  $k$  is affected by  $(u, v)$  but not by  $u$ 's existing transmissions.

We present PRP and the *WEIGHT* function in Algorithms 3 and 4, respectively. In Lines 13 through 17 in Algorithm 3, new cumulative receiving power is computed for each node. In Lines 18 through 20, the sender's cumulative transmitting power is calculated. In a TPG consisting of  $n$  vertices and  $m$  arcs, the *while* loop in PRP, each

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**Algorithm 2** Cumulative Designated Receiver Power (CDRP)

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**Input:** A TPG  $G = (V, A, p)$  and a node  $s \in V$   
**Output:** A spanning tree  $T = (V', A')$

- 1:  $V' \leftarrow \{s\}; A' \leftarrow \phi$
- 2: **for all**  $u \in V$  **do**
- 3:    $p_u^{rc} \leftarrow 0, p_u^{tr} \leftarrow 0$
- 4: **end for**
- 5: **while**  $|V'| < |V|$  **do**
- 6:    $w(u^0, v^0) \leftarrow \infty$   $\{(u^0, v^0)$  is the link to be included in  $T\}$
- 7:   **for all**  $u \in V'$  **and**  $v \in V \setminus V'$  **and**  $(u, v) \in A$  **do**
- 8:     **if**  $w(u^0, v^0) > \max\{p_u^{ce} + p(u, v) + p_u^{rc}, \hat{p}_v^{rc}\}$  **then**
- 9:        $u^0 \leftarrow u; v^0 \leftarrow v$
- 10:     **end if**
- 11:   **end for**
- 12:    $V' \leftarrow V' \cup \{v\}; A' \leftarrow A' \cup \{(u^0, v^0)\}$
- 13:   **for all**  $k \in V$  **and**  $(u, k) \in A$  **and**  $p(u, k) \leq p(u, v)$  **do**
- 14:     **if**  $p(u, k) > p_u^{tr}$  **then**
- 15:        $p_k^{rc} \leftarrow p_k^{rc} + \hat{p}_k^{rc}$  { update receiving power if  $k$  was not reached by  $u$  before }
- 16:     **end if**
- 17:   **end for**
- 18:   **if**  $p_u^{tr} < p(u, v)$  **then**
- 19:      $p_u^T \leftarrow p(u, v)$
- 20:   **end if**
- 21: **end while**
- 22: **return**  $T$

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for loop in PRP, and the *for* loop in the *WEIGHT* function run  $n$ ,  $O(m)$  and  $O(n)$  times, respectively. Therefore, the time complexity of PRP is  $O(mn^2)$ .

It is worth noting that each of the four algorithms performs the same under the NOC model. This is because  $\bar{p}_u^{rc} = p_u^{rc} = 0$  for each node  $u$  in the network and the weight of a link in a TPG is reduced to its TPT. Under the DRP model, where  $\bar{p}_v^{rc} = 0$  for any node  $u$  if  $v$  is not a designated receiver of  $u$  in the resulting graph, we can easily verify that the weight of a link defined by (2) and (3) have the same value as defined by (1) and hence, CDRP and PRP are the same as DRP.

## 5 Simulation Results

In this section, we report the simulation results. We simulated networks consisting 20 to 100 nodes randomly deployed in a  $100 \times 100 m^2$  field. The power consumption model and the battery model described in Section 3 were adopted. The maximum transmission power was set to 1000 mW for all the nodes. We assumed an identical level of battery energy for each node. The broadcast tree lifetime is measured in terms of the maximum nodal power consumption, which is inversely proportional to the broadcast tree lifetime according to the discussion in Section 3.3. In figures which record the maximum power consumption, each data point averages the results in 100 random deployments and is depicted with 95% confidence interval.

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**Algorithm 3** Proximity Receiver Power (PRP)

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**Input:** A TPG  $G = (V, A, p)$  and a node  $s \in V$   
**Output:** A spanning tree  $T = (V', A')$

- 1:  $V' \leftarrow \{s\}; A' \leftarrow \phi$
- 2: **for all**  $u \in V$  **do**
- 3:    $p_u^{rc} \leftarrow 0, p_u^{tr} \leftarrow 0$
- 4: **end for**
- 5: **while**  $|V'| < |V|$  **do**
- 6:    $w(u^0, v^0) \leftarrow \infty$   $\{(u^0, v^0)$  is the link to be included in  $T\}$
- 7:   **for all**  $u \in V'$  **and**  $v \in V \setminus V'$  **and**  $(u, v) \in A$  **do**
- 8:     **if**  $w(u^0, v^0) > WEIGHT(i, j)$  {See Algorithm 4} **then**
- 9:        $u^0 \leftarrow u, v^0 \leftarrow v$
- 10:     **end if**
- 11:   **end for**
- 12:    $V' \leftarrow V' \cup \{v\}; A' \leftarrow A' \cup \{(u, v)\}$
- 13:   **for all**  $k \in V$  **and**  $(u, k) \in A$  **and**  $p(u, k) \leq p(u, v)$  **do**
- 14:     **if**  $p(u, k) > p_i^{tr}$  **then**
- 15:        $p_k^{rc} \leftarrow p_k^{rc} + \hat{p}_k^{rc}$
- 16:     **end if**
- 17:   **end for**
- 18:   **if**  $p_u^{tr} < p(u, v)$  **then**
- 19:      $p_u^{tr} \leftarrow p(u, v)$
- 20:   **end if**
- 21: **end while**
- 22: **return**  $T$

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We assigned symmetric or asymmetric transmission power thresholds for the two links connecting a pair of nodes in opposite directions. In the symmetric setting, for nodes  $u$  and  $v$ , we had  $p(u, v) = p(v, u) = \frac{d^2}{2}$ , where  $d$  (in meters) is the Euclidean distance between  $u$  and  $v$  if  $p(u, v)$  is less than the maximum transmission power; in the asymmetric setting, we randomly selected either  $p(u, v) = d^2$  or  $p(u, v) = \frac{d^2}{2}$  independent upon  $p(v, u)$ . We set  $p_u^{ce} = 0$  for each node  $u$  for simplicity. In order to guarantee fairness, this selection was recorded and applied to each algorithm.

We simulated scenarios where the receiving power is 0. Fig. 3 shows these simulation results. The four curves perfectly overlap because each one is optimal under the NRP model. The maximum power consumption reduces as the network size increases. This is because the average transmission power level decreases in a denser network. With the same network size, each data point with asymmetric transmission power has a somewhat bigger value than the one with symmetric transmission power. This is because we assigned smaller average transmission power thresholds in the symmetric transmission power setting than that in the asymmetric setting.

We also simulated scenarios where the receiving power level is a non-zero constant. Fig. 4 shows these simulation results. In this set of simulations, the receiving power was set to half of the maximum transmission power level, which is 500 mW, for all the nodes. We also ran simulations with different receiving power values. These results showed similar trends. It is clear that PRP outperforms TPO, DRP and CDRP. While

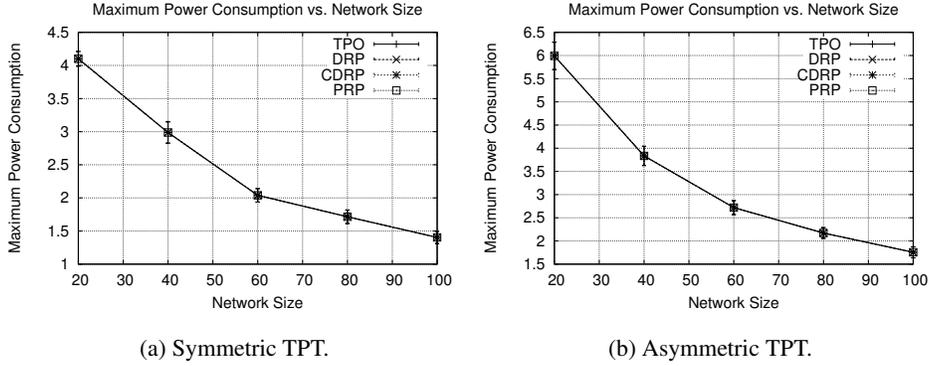
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**Algorithm 4** Function:  $WEIGHT(u, v)$ 

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```
1:  $weight \leftarrow p_u^{te} + p(u, v) + p_u^{rc}$ 
2: for all  $k \in V$  and  $p(u, k) \leq p(u, v)$  do
3:   if  $weight < p_k^{ce} + p_k^{tr} + p_k^{rc} + \hat{p}_k^{rc}$  and  $p(u, k) > p_u^{tr}$  then
4:      $weight \leftarrow p_k^{ce} + p_k^{tr} + p_k^{rc} + \hat{p}_k^{rc}$ 
5:   else if  $weight < p_k^{ce} + p_k^{tr} + p_k^{rm}$  and  $p(u, k) \leq p_u^{tr}$  then
6:      $weight \leftarrow p_k^{ce} + p_k^{tr} + p_k^{rc}$ 
7:   end if
8: end for
9: return  $weight$ 
```

---



**Fig. 3.** No Receiving Power.

the other 3 curves increase linearly as the network becomes denser, the one due to PRP is almost constant. We can also see that CDRP outperforms TPO and DRP, although the difference is not as pronounced. The difference between symmetric and asymmetric transmission power setting is evident because the transmission power level does not dominate.

We simulated scenarios where the receiving power is node-dependent. Fig. 5 shows these simulation results. In this set of simulations, for each node, the receiving power was randomly selected from  $\frac{1}{3}$ ,  $\frac{1}{4}$  and  $\frac{1}{5}$  of the maximum transmission power level. Other various receiving power settings gave us results similar to the one depicted in Fig. 5. Again, PRP evidently outperforms all the other solutions. The advantage of CDRP decreases even further.

## 6 Conclusions

In this paper, we investigated the MaxBTL problem under the OC model that takes into account the overhearing cost. While it is polynomially solvable under the NOC model, it becomes NP-hard under the OC model. We summarized the two optimal solutions,

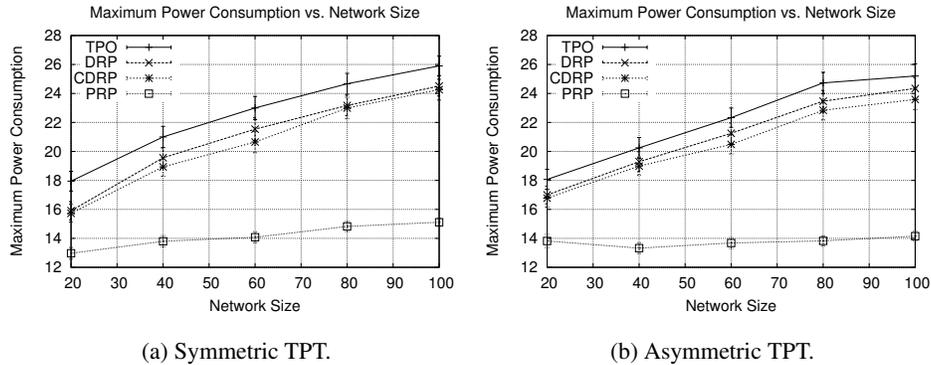


Fig. 4. Identical Receiving Power.

TPO and DRP, proposed to the problem of MaxBTL under the NOC model and showed that they can have an approximation ratio as bad as  $\Omega(n)$  under the OC model. We proposed two greedy heuristics solutions, CDRP and PRP, by considering the effects of overhearing cost when generating a broadcast tree. Simulation results showed that our solutions outperformed existing solutions. In particular, PRP performed better than TPO and DRP by up to 100%.

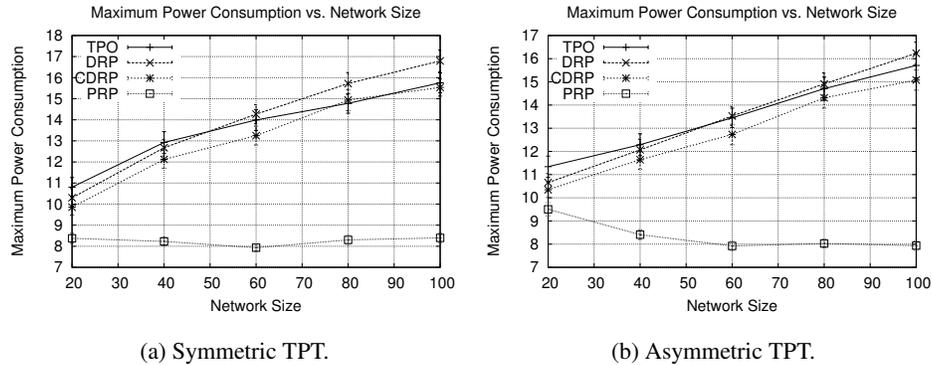
In this paper, the approaches were mainly proposed to tackle a broadcast problem. In the case of multicast, one solution is to prune the resulting broadcast tree such that only the necessary nodes for the multicast session are included. Performance evaluation of this prune-based approach is taken as future work.

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## References

1. Lloyd, E.L., Liu, R., Marathe, M.V.: Algorithm aspects of topology control problems for ad hoc networks. In: Proc. ACM MOBIHOC, EPFL lausanne, Switzerland (2002)
2. Floréen, P., Kaski, P., Kohonen, J., Orponen, P.: Multicast time maximization in energy constrained wireless networks. In: DIALM-POMC, San Diego, CA (2003)
3. Kang, I., Poovendran, R.: Maximizing static network lifetime of wireless broadcast adhoc networks. In: Proc. IEEE ICC. Volume 3. (2003)
4. Das, A.K., Marks, R.J., El-Sharkawi, M., Arabshahi, P., Gray, A.: MDLT: a polynomial time optimal algorithm for maximization of time-to-first-failure in energy constrained wireless broadcast networks. In: Proc. IEEE GLOBECOM. (2003)
5. Deng, G., Gupta, S.: Maximizing broadcast tree lifetime in wireless ad hoc networks. In: Proc. IEEE GLOBECOM. (2006)



**Fig. 5.** Non-identical Receiving Power.

6. Floréen, P., Kaski, P., Kohonen, J., Orponen, P.: Lifetime maximization for multicasting in energy-constrained wireless networks. *IEEE J. Select. Areas Commun.* **23**(1) (2005)
7. Kang, I., Poovendran, R.: Maximizing network lifetime of broadcasting over wireless stationary ad hoc networks. *ACM/Kluwer MONET Special Issue on Energy Constraints and Lifetime Performance in Wireless Sensor Networks* **10**(6) (2005) 879–896
8. Park, S., Savvides, A., Srivastava, M.: Battery capacity measurement and analysis using lithium coin cell battery. In: *ISLPED*, New York, NY, USA, ACM Press (2001)
9. Cui, S., Goldsmith, A.J., Bahai, A.: Modulation optimization under energy constraints. In: *Proc. IEEE ICC.* (2003)
10. Basu, P., Redi, J.: Effect of overhearing transmissions on energy efficiency in dense sensor networks. In: *Proc. ACM the third international symposium on Information processing in sensor networks.* (2004)
11. Vasudevan, S., Zhang, C., Goeckel, D., Towsley, D.: Optimal power allocation in wireless networks with transmitter-receiver power tradeoffs. In: *Proc. IEEE INFOCOM.* (2006)
12. Camerini, P.M.: The min-max spanning tree problem and some extensions. *Information Processing Letters* **7**(1) (1978) 10–14
13. Burkhart, M., von Rickenbach, P., Wattenhofer, R., Zollinger, A.: Does topology control reduce interference? In: *Proc. ACM MOBIHOC.* (2004)
14. Moaveni-Nejad, K., Li, X.Y.: Low-interference topology control for wireless ad hoc networks. *Ad Hoc & Sensor Wireless Networks* (2005)
15. Wieselthier, J.E., Nguyen, G.D., Ephremides, A.: On the construction of energy-efficient broadcast and multicast trees in wireless networks. In: *Proc. IEEE INFOCOM.* (2000)
16. Raghunathan, V., Schurgers, C., Park, S., Srivastava, M.B.: Energy-aware wireless microsensor networks. In: *IEEE Signal Processing Mag. Volume 19.* (2002)
17. Rakhmatov, D., Vrudhula, S., Wallach, D.A.: A model for battery lifetime analysis for organizing applications on a pocket computer. *IEEE Trans. VLSI Syst.* **11**(6) (2003) 1019–1030
18. Chiasserini, C.F., Rao, R.R.: Energy efficient battery management. *IEEE J. Select. Areas Commun.* **19**(7) (2001) 1235–1245
19. Adamou, M., Sarkar, S.: A framework for optimal battery management for wireless nodes. In: *Proc. IEEE INFOCOM.* (2002)
20. GAREY, M.R., JOHNSON, D.S.: *Computers and Intractability, A Guide to the Theory of NP-Completeness.* W. H. Freeman and Company, New York, NY (1979)