Center at Nearest Source.
Euclidean geometry:

1. All points are vectors in $d$-dimensional space.

   \[ \mathbf{v} : (v_1, v_2, \ldots, v_d) \]

   \[ d = 2 \text{ or } 3 \]

   \[ d=2 \quad \mathbf{v} : (v_x, v_y) \]

   \[ |\mathbf{v}| = \sqrt{v_x^2 + v_y^2} \]

   \[ d=3 \quad \mathbf{v} : (v_x, v_y, v_z) \]

   \[ |\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} \]

   \[
   \text{dist}(p, q) = |p - q|
   = \sqrt{(p_x - q_x)^2 + (p_y - q_y)^2 + (p_z - q_z)^2}
   \]
Euclidean MST Problem:
Given a set of points P, compute
the MST in the complete weighted
graph of P.
Let's call this complete
graph G.

\[ G^2 \quad \text{weights are square of Euclidean distance} \]
\[ G^3 \quad \text{weights are} \ \text{dist}^2(\cdot, \cdot) \]
So if the path-loss constant is $\alpha$, then the shortest-weight path is $G^*$ represents the minimum energy path between two nodes.

\[
\begin{align*}
    P_r &= \frac{P_t}{d^\alpha} \\
    P_{th} \cdot d^\alpha &= P_{T_{min}} \\
    P_{T_{min}} &\propto d^{-\alpha} \text{ path-loss constant}
\end{align*}
\]
**Naive Algorithm for Constructing Euclidean MST:**

1. Construct $G^*$
   - $O(n^2)$

2. Use Prim's algorithm to construct MST
   - $O(n \log n)$

$n \log n$ algorithm uses Delaunay triangulation.

Voronoi Cells

Voronoi

Diagram of Delaunay triangulation and Voronoi cells.
Is Euclidean MST equivalent to minimum energy spanning tree to cover a set of points in space?

\[ a = b + 5 \]

\[ 2a + b = \pi \]

\[ a > b \quad \frac{b}{a} < \frac{\pi}{3} \]
Cost_{EMST} = 6

Optimum Cost to cover all the nodes (P₁, . . . , Pₙ) from 0 is to just do one transmission of cost 1 + ε. This assumes omnidirectional antenna. This is due to wireless transmission property.

For this particular example, called wireless multicasting advantage:

\[
\frac{\text{Cost}_{EMST}}{\text{Cost}_{Opt}} \geq \frac{6}{1 + \epsilon}
\]
$\epsilon \to 0$

\[
\frac{\text{Cost EMST}}{\text{Cost Opt.}} \approx 6.
\]

\[
6 \leq \text{Approx. Ratio} \leq 12 \\
\text{for EMST}
\]

for computing the minimum energy broadcast tree problem.
Energy efficiency versus lifetime.

- Battery capacity: B
- Each packet consumes certain energy P for transmission.
- # of packets that can be transmitted = \( \frac{B}{P} \) ≤ lifetime of a node in terms of how many packets it can transmit.

Minimum energy tree cost is 10.

A has better lifetime than minimum energy tree B.

200 > 250

\[ \frac{200}{10} = 20 \]