Q1

2. \( ND \leq NA \)

3. \( ND \leq (K + 1)X + \min(d_i) \)
   \( NA = \sum d_i \)
   \( X \leq \min(d_i) \)
4. 

\[ 4 = 2 + 2 \]

3
Suboptimal tree construction methods.

1. Center at Nearest Source.
2. Shortest Path Tree.
3. Greedy

Incremental Tree (GIT)

1. Initially the sink node is the only node in the tree $T$ (correct)
2. At each step add the closest source node to $T$
Shortest Path Tree

1. For each source $S_i$, find the shortest path $P_i$ to sink node $S_i$

2. Combine, path $P_i$ & $P_j$ if they meet at some intermediate point.
**Theorem**

If subgraph $G'$ induced by the source nodes is connected then the optimal tree can be formed in polynomial time.

$G : V, E$

$G' : V_{es_1}, \ldots, V_{es_k}$

$e, e \in E$ and both the endpoints are in $V'$

$G'$ is induced graph by set $V'$
Proof: Consider GIT

1. 1st source connected to tree will be at \( \min(d_i) \) distance

2. at each remaining \( k-1 \) steps
   - one source can be added at the cost of 1 additional transmission
Total tree cost = \text{min}(d_i) + K - 1 \text{ transmissions} = \text{lower bound on the cost of} \text{ DC tree} \text{ hence is optimal}

\text{GIT is polynomial time algorithm since it is } O(K.n^2)

\text{Corollary Theorem} \quad \text{In the ER model, when } R > 25 \text{, the optimal data aggregation tree can be formed in polynomial time}
MST - Minimum-weight spanning tree
- Each edge $e$ has weight $w(e)$ associated with it.
- $W(T) = \sum_{e \in T} w(e)$, $T$ is a tree.
- A minimum weight spanning tree (MST) is a spanning tree with minimum possible weight among all spanning trees.
An interesting property of MST...

Partition A

Partition B

link with minimum weight among $e_1, \ldots, e_k$ is guaranteed to be $e_i$.

Then $\exists$ a MST with $e_i$ in it.